Math-1.1

Sl.No. :

Total No. of Pages : 2

First Semester M.Sc. Degree Examination, May - 2025

(SLM Scheme) (Non - CBCS)

MATHEMATICS

Algebra

Time : 3 Hours **Max. Marks : 80**

Instructions : 1) Answer any five full questions.

2) All questions carry equal marks.

Q1) a) Define a subgroup of a group. Let H and K be subgroup of a group G. Then prove that HK is a subgroup of G, if and only if HK= KH.

b) State and prove Lagrange theorem for finite groups.

c) State and prove the fundamental theorem of homomorphism of groups.

[6 + 5 + 5]

Q2) a) Derive the class equation for finite groups.

b) Prove that a group of order p^n, $n \geq 1$ where P is a prime has non trivial center.

c) Prove that a group of order $5^2.7^2$ is abelian.

[6 + 4 + 6]

Q3) a) Define an integral domain. Show that every integral domain can be embedded in a field.

b) Prove that a commutative ring R with identity is a field if and only if the only ideals of R are {0} and R itself.

[10 + 6]

Q4) a) State and prove the fundamental theorem of homomorphism for rings.

b) Let R be a commutative ring with identity. Then prove that an ideal M is a maximal ideal in R if and only if R/M is a field.

c) Let R be an integral domain and R[X] be set of all polynomials over R. Then prove that every unit in R[X] is a unit in R.

[8 + 4 + 4]

Q5) a) Prove that a nonzero element P of a ring R is irreducible if and only if (P) is maximal among all proper principal ideals of R.

b) Prove that in a unique factorization domain, an element is prime if and only if it is irreducible.

c) If F is a field then prove that F[X] is an euclidean domain.

[4 + 8 + 4]

Q6) a) State and prove Eisenstein's criterion.

b) If F, K and L are fields such that K is algebraic over F and L is algebraic over K then prove that L is algebraic over F.

[8 + 8]

Q7) a) Let $f(x) \in F[X]$ be of degree n. Then prove that there is an extension K of F which is a splitting field of $f(X)$ and the degree of K over F is atmost n!

b) Find the splitting field of X^3-2 over Q

c) Let $f(X) \in F(X)$ be an irreducible polynomial. Then prove that $f(X)$ has a multiple root if and only if $Df(X)$ is a Zero Polynomial.

[8 + 4 + 4]

Q8) a) Prove that the multiplicative group of non-zero elements of a finite field is cyclic.

b) State and prove the primitive element theorem.

[6 + 10]

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