Math-1.2

Sl.No. :

Total No. of Pages : 2

First Semester M.Sc. Degree Examination, May/June - 2023

(Non CBCS) (SLM Scheme)

MATHEMATICS

Real Analysis - I

Time : 3 Hours **Max. Marks : 80**

Instructions : 1) Answer any five full questions.

2) All questions carry equal marks.

Q1) a) Prove that the set of all rational numbers does not have the least upper bound property.

b) Prove that for every real $x > 0$ and every integer $n > 0$. There is one and only one real $y > 0$ such that $y^n = x$.

[8 + 8]

Q2) a) Define countable set prove that every sub set of a countable set is countable.

b) Let A be the set of all sequences whose elements are the digits 0 and 1. Then prove that A is uncountable.

[8 + 8]

Q3) a) Define metric space. Prove that a set E in a metric space X is open if and only if its complement is closed.

b) Let X be a metric space and $E \subset X$. Then prove that

i) $\overline{E}$ is closed

ii) $E = \overline{E}$ if and only if E is closed

iii) $\overline{E} \subset F$ for every closed set $F \subset X$ such that $E \subset F$.

[8 + 8]

Q4) a) Suppose K $\subset$ Y $\subset$ X. Then prove that K is compact relative to X if and only if K is compact relative to Y.

b) Prove that the closed interval $[a, b]$ is compact in R'.

c) Prove that sequence $\{x_n\}$ converges to x if and only if every subsequence of $\{x_n\}$ converges to x.

[5 + 6 + 5]

Q5) a) Prove that the subsequential limits of a sequence $\{x_n\}$ form a closed subset of $\mathbb{R}$.

b) Prove that $\lim_{n\to\infty} \sqrt[n]{p} = 1$, if $p > 0$.

c) Suppose $\{x_n\}$ is monotonic then prove that $\{x_n\}$ converges if and only if it is bounded.

[6 + 4 + 6]

Q6) a) State and prove Bolzano-Weirstrass theorem.

b) Examine the following series for convergence :

i) $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ ii) $\sum_{n=1}^{\infty} \frac{2n+1}{3n+2}$

[8 + 8]

Q7) a) Prove that the series $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^p}$ converges if P > 1 and diverges if P $\leq$ 1.

b) State and prove D' Alembert's ratio test.

[8 + 8]

Q8) a) State and prove Abel's test.

b) State and prove Kummer's test.

[8 + 8]

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