Math-1.2

Sl.No. :

Total No. of Pages : 2

First Semester M.Sc. Degree Examination, May - 2025

(SLM Scheme) (Non - CBCS)

MATHEMATICS

Real Analysis - I

Time : 3 Hours **Max. Marks : 80**

Instructions : 1) Answer any five full questions.
2) All questions carry equal marks.

Q1) a) Prove that the set of all rational numbers does not have the least upper bound property.

b) Let $a \geq 0$ then prove that $|x| \leq a$ if and only if $-a \leq x \leq a$.

c) Prove that every subset of a countable set is countable.

[7 + 4 + 5]

Q2) a) Prove that the set of all rational numbers is countable.

b) Prove that set (0,1) is uncountable.

c) Show that $(\mathbb{R}, d)$ is a metric space where $d : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is given by $d(x, y) = |x - y|$.

[6 + 6 + 4]

Q3) a) Prove that the set E in a metric space X is open if and only if its complement is closed.

b) Let X be a metric space and $Y \subset X$. Show that a subset E of Y is open relative to Y if and only if $E = Y \cap G$ for some open subset G of X.

c) Prove that compact subsets of metric spaces are closed.

[5 + 6 + 5]

Q4) a) Prove that the subsequential limits of a sequence $\{x_n\}$ forms a closed subset of $\mathbb{R}$.

b) If $\{x_n\}$ and $\{y_n\}$ are sequences with $\lim_{n\to\infty} x_n = x$ and $\lim_{n\to\infty} y_n = y$ then prove that $\lim_{n\to\infty} x_n y_n = xy$.

c) If $p > 0$ then prove that $\lim_{n\to\infty} \sqrt[n]{n} = 1$.

[8 + 4 + 4]

Q5) a) If $\{x_n\}$ and $\{y_n\}$ are sequences of real numbers such that $x_n \le y_n$, $\forall_n$, then prove that $\lim_{n\to\infty} \sup x_n \le \lim_{n\to\infty} \sup y_n$.

b) State and prove the Cauchy criterion for convergence of a sequence.

[8 + 8]

Q6) a) Prove that $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$. Also show that e is irrational.

b) State and prove Bolzano-Weirstrass theorem.

[8 + 8]

Q7) a) State and prove Cauchy's first limit theorem.

b) Prove that a series of non-negative terms is convergent if and only if its sequence of partial sums is bounded.

c) Examine the following series for convergence :

i) $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$

ii) $\sum_{n=0}^{\infty} \frac{1}{n!}$

[8 + 4 + 4]

Q8) a) Suppose $\sum_{n=1}^{\infty} a_n$ converges and $\{b_n\}$ is monotonic and bounded. Then prove that $\sum_{n=1}^{\infty} a_n b_n$ converges.

b) State and prove Merten's theorem.

[8 + 8]

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