Math-1.3

Sl.No. :

Total No. of Pages : 2

First Semester M.Sc. Degree Examination, May - 2025

(SLM Scheme) (Non-CBCS)

MATHEMATICS

Complex Analysis - I

Time : 3 Hours **Max. Marks : 80**

Instructions : 1) Answer any five full questions.

2) All questions carries equal marks.

Q1) a) State and prove Lagrange's identity in the complex form.

b) Prove that $\left|\frac{Z_1 - Z_2}{1 - \bar{Z}_1 Z_2}\right| = 1$ if either $|Z_1| = 1$ of $|Z_2| = 1$. What exception must be made if $|Z_1| = |Z_2| = 1$?

[8 + 8]

Q2) a) Find the distance between two points on Reimann sphere.

b) Let Z_1 and Z_2 be any two complex numbers, then prove that arg $(Z_1 Z_2) = \arg Z_1 + \arg Z_2$.

c) Prove that the centroid of the triangle whose vertices are Z_1, Z_2, Z_3 is $\frac{1}{3}(Z_1 + Z_2 + Z_3)$.

[8 + 4 + 4]

Q3) a) If Z_1, Z_2, Z_3 are the vertices of an isosceles right angled triangle at the vertices Z_2, prove that $Z_1^2 + 2Z_2^2 + Z_3^2 = 2Z_2(Z_1 + Z_3)$.

b) Find the general solution of a straight line.

c) Show that if the equation $Z^2 + \alpha Z + \beta = 0$ has a pair of conjugate complex roots, then α, β are both real and $\alpha^2 < 4\beta$.

[6 + 6 + 4]

Q4) a) State and prove necessary and sufficient condition for a function to be analytic.

b) Show that an analytic function with constant modulus is constant.

[12 + 4]

Q5) a) Prove that a sequence $\{Z_n\}$ is convergent if and only if the real sequences $\{\mathrm{Re}\, Z_n\}$ and $\{I_m\, Z_n\}$ are both converged and also

$$\lim_{n\to\infty} Z_n = \lim_{n\to\infty} (\mathrm{Re}\, Z_n) + i \lim_{n\to\infty} (I_m\, Z_n)$$

b) State and prove Weierstrass M-test for uniform convergence of a series of functions.

[8 + 8]

Q6) a) If $E(Z) = \sum_{n=0}^{\infty} \frac{Z^n}{n!}$ then prove that $E(Z_1 + Z_2) = E(Z_1)\, E(Z_2)$.

b) Show that the half plane $x > 0$ maps onto the half plane $v > 1$ under the transformation $iZ + i$.

c) Prove that any bilinear transformation preserves cross ratio.

[4 + 4 + 8]

Q7) a) If $f(z)$ is analytic in the entire complex plane and bounded, then show that $f(z)$ is a constant function.

b) State and prove fundamental theorem of algebra.

c) Evaluate $\int_C \frac{Z-3}{Z^2-2Z+5} dz$, where C is the circle $|Z + 1 - i| = 2$.

[6 + 6 + 4]

Q8) a) State and prove Cauchy's integral formula.

b) Suppose that f is analytic in a domain D and 'a' is a point in D such that $|f(z)| \le |f(a)|$ holds for all $z \in$ D. Then prove that f is constant.

[8 + 8]

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