Math-1.4

Sl.No. :

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First Semester M.Sc. Degree Examination, May/June - 2023

(Non CBCS) (SLM Scheme)

MATHEMATICS

Discrete Mathematics

Time : 3 Hours **Max. Marks : 80**

Instructions : 1) Answer any five full questions.

2) All questions carry equal marks.

Q1) a) Define logical connectives. Construct the truth tables for the following statements.

i) Distributive laws

ii) De' Morgans laws

b) Define : Disjunctive Normal Form (DNF);

Conjunctive Normal Form (CNF); Principal

Disjunctive Normal Form (PDNF) and Principal

Conjunctive Normal Form (PCNF).

Obtain DNF and CNF of $p \wedge (p \rightarrow q)$.

[8 + 8]

Q2) a) Establish the following result using the principles of mathematical induction.

$$1^2 + 3^2 + 5^2 + ... + (2n-1)^2 = \frac{n\left(4n^2 - 1\right)}{3}.$$

b) State and prove the pigeonhole principle and extended pigeonhole principle.

c) How many 8 digit telephone number can have one or more repeated digits?

[6 + 6 + 4]

Q3) a) In how many ways can 6 men and 6 women can be seated in rows so that,

i) If any person may sit next to any other.

ii) If men and women must occupy alternatively.

b) Find the number of non-negative integers of the equality.

$x_1 + x_2 + x_3 + x_4 + x_5 = 30$ with $x_1 \geq 2$; $x_2 \geq 3$; $x_3 \geq 4$; $x_4 \geq 2$ and $x_5 \geq 0$.

c) Find the term which contain x'' and y^4 in $(3x - 3y^2 + 2z^2)6$.

[5 + 5 + 6]

Q4) a) Solve the following recurrence relation :

i) $a_n + 4a_{n-1} + 4a_{n-2} = 8$ given $a_0 = 1$, $a_1 = 2$.

ii) $a_{n+2} - 10a_{n+1} + 21a_n = 3n^2 - 2$ for $n > 0$ given $a_0 = 1$, $a_1 = 2$.

b) Solve the recurrence relation $u_{n+2} + u_{n+1} + u_n = n^2 + n + 1$ using difference method.

[8 + 8]

Q5) a) State and prove the Towers of Hanoi Problem.

b) Solve the recurrence relation. $F_{n+2} = F_{n-1} + F_n$ for $n \geq 0$, given $F_0 = 0$, $F_1 = 1$.

[8 + 8]

Q6) a) Define : Reflexive relation, symmetric relation, transitive relation, irreflexive relation and antisymmetric relation. Give an example of a relation which is both symmetric and antisymmetric.

b) If $R = \{(x, y)/x - y$ is divisible by $3\}$. The show that R is an equivalence relation on the set of integers $\mathbb{Z}$.

c) Let R be an equivalence relation defined on a set A. Then prove the following:

i) Each equivalence is non-empty and A is the union of all equivalence class.

ii) $aRb \Leftrightarrow [a] = [b]$

iii) Any two equivalence class are either identical or disjoint.

[5 + 3 + 8]

Q7) a) Define connectivity relation R^{∞} on R. Prove that for a relation R on a set A. R^{∞} is the transitive closure of R.

b) If A is a set with |A| = n and R is a relation on A. Then prove that $R^{\infty} = RUR^2U \ldots\ldots UR^n$.

[8 + 8]

Q8) a) Define partial ordering set. Draw the Hasse diagram representing the positive divisors of 36.

b) Define Lattice with an example. Let (L, $\leq$) be a lattice. Then show that

$$a \leq b \Rightarrow \begin{cases} a \vee c \leq b \vee c \\ a \wedge c \leq b \wedge c \end{cases}, \forall a, b \in \mathrm{L}.$$

[8 + 8]

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