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I Semester M.Sc. Degree Examination, May - 2025

(SLM Scheme) (Non-CBCS)

MATHEMATICS

Discrete Mathematics

Time : 3 Hours **Max. Marks : 80**

Instructions : 1) Answer any five full questions.
2) All questions carry equal marks.

Q1) a) Define logical connectives construct the truth table for the following statements.

i) $(p \to q) \leftrightarrow (\sim p \vee q)$

ii) $p \wedge (q \vee r) \to (p \wedge q) \vee (p \wedge r)$

b) Define Disjunctive Normal Form (DNF), Conjunctive Normal Form (CNF). Obtain CNF of the following statements.

i) $p \wedge (p \to q)$

ii) $\sim (p \vee q) \leftrightarrow (p \wedge q)$

[8 + 8]

Q2) a) Obtain Principal Disjunctive Normal Form (PDNF) of the given statements.

i) $(\sim p \vee \sim q) \to (\sim p \wedge r)$

ii) $p \to [(p \to q) \wedge \sim(\sim q \vee \sim p)]$

b) By mathematical induction prove that for all positive integers $n \geq 1$,

$1^3 + 2^3 + ... + n^3 = \dfrac{n^2(n+1)^2}{4}$.

c) Negate the following statement :

"If some straight line intersect, then all straight line are not parallel."

[8 + 4 + 4]

Q3) a) State and prove the well - ordering principle.

b) State and prove Pigeonhole principle and Extended Pigeonhole principle.

c) Prove that if seven numbers from 1 to 12 are chosen then two of them will add upto 13.

[5 +8 + 3]

Q4) a) In how many ways can 6 men and 6 women can be seated in rows so that

i) If any person may sit next to any other

ii) If men and women must occupy alternatively

b) Find the number of non-negative integers of the equality
$x_1 + x_2 + x_3 + x_4 + x_5 = 30$ with $x_1 \geq 2$, $x_2 \geq 3$, $x_3 \geq 4$, $x_4 \geq 2$ and $x_5 \geq 0$.

c) Find the Co-efficient of x^{12} in $x^3(1 - 2x)^{10}$.

[6 + 6 + 4]

Q5) a) Explain the rule of solving first order recurrence relation.

b) Solve a recurrence relation for the number of binary sequence of length $n \geq 1$ that have no consecutive zero's.

c) Solve the recurrence relation $a_{n+1} - a_n = 3^n$, for $n \geq 0$ and $a_0 = 1$, by using generating function.

[4 + 8 + 4]

Q6) a) State and prove the Towers of Hanoi problem.

b) Find the generating function for the recurrence relation : $a_{n+1} - a_n = n^2$, $n \geq 0$ and $a_0 = 1$. Hence solve the relation.

[8 + 8]

Q7) a) State and prove Fundamental theorem of an equivalence relation.

b) Define connectivity relation R^{∞} on R. Prove that for a relation R on a set A, R^{∞} is the transitive closure of R.

[8 + 8]

Q8) a) Define partial ordering set. Draw the Hasse diagram representing the positive divisors of 36.

b) Define Lattice with an example. Let (L, $\leq$) be a lattice. Then show that

$$a \leq b \Rightarrow \begin{cases} a \vee c \leq b \vee c \\ a \wedge c \leq b \wedge c \end{cases}, \forall\, a,b,c \in L .$$

[8 + 8]

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