Math-1.5

Sl.No. : **Total No. of Pages : 2**

First Semester M.Sc. Degree Examination, May/June - 2023
(NON CBCS) (SLM Scheme)
MATHEMATICS
Differential Equations

Time : 3 Hours **Max. Marks : 80**

Instructions : 1) Answer any five full questions.
2) No addition sheets will be provided for answering.
3) Use of scientific calculator is permitted.

1) a) Define fundamental set of solutions of a differential equation. State and prove necessary and sufficient condition for $\{\phi_j(x) : j = 1,2, ..., n\}$ to form a fundamental set of solutions of n^{th} order linear differential equation $L_n y = 0$.

b) State and prove Liouville's theorem.

[8+8]

2) a) Find the Wronskian of the independent solutions of $y^{(v)} - y^{(iv)} - y' + y = 0$ in [0.1]

b) Define adjoint of a n^{th} order linear differential equation $L_n y = 0$. Find the adjoint differential equation to $x^2 y'' + 7xy' + 8y = 0$.

c) Solve by the method of variation of parameters $x^2 y'' - xy' - 3y = x^3$.

[4+8+4]

3) a) State and prove Sturm's separation theorem.

b) Find the eigen values and eigen functions of starm Liouville's problem $y'' + \lambda y = 0$ $y(0) = y(\pi) = 0$.

[10+6]

4) a) Find the series solution of the equation $y'' + xy' + (x^2+2) y = 0$.

b) Obtain generating function for Laguerre polynomial.

[10+6]

5) a) Find the general solution of following homogeneous linear equation

$$\frac{dx}{dt} = \begin{pmatrix} 7 & -1 & 6 \\ -10 & 4 & -12 \\ 2 & 1 & -1 \end{pmatrix} X$$

b) Define critical point, trajectorics, saddle point, stable and unstable critical points of a system of differential equations.

[10+6]

6) a) Solve the following Cauchy problem $u_x + u_y + u - 1 = 0$ with the initial data $u = \sin x$ on $y = x + x^2$.

b) Describe the classification of first order partial differential equations with examples.

[10+6]

7) a) Define complete integral of a non-linear first order partial differential equation. Find the complete integral of the equation $pq - p - q = 0$.

b) Explain Jacobi's method of finding complete integral of partial differential equation. Using the method find the complete integral of $u_x x^2 - u_y^2 - a u_z^2 = 0$.

[8+8]

8) a) Classify the equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + 2\frac{\partial^2 u}{\partial y \partial x}$ and reduce it to Cannonical form.

b) Explain the Cannonical transformation of solving second order partial differential equation.

[8+8]

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