# Math-1.5

Sl.No. :

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## First Semester M.Sc. Degree Examination, May - 2025
## (SLM Scheme) (Non CBCS)
## MATHEMATICS
## Differential Equations

**Time : 3 Hours** **Max. Marks : 80**

***Instructions :*** ***1) Answer any five full questions.***
***2) All questions carry equal marks.***
***3) Use of scientific calculator is permitted.***

***Q1)*** a) State the existance and uniqueness theorem for an $n^{th}$ order initial value problem. Find the solution of the initial value problem $y' = 2x$, $y(0) = 1$. Show that the solution obtained is unique.

b) Find solution $\phi_1(x)$ and $\phi_2(x)$ of $y'' + k^2y = 0$ subject to initial conditions $\phi_1(0) = \sin\alpha$, $\phi_1^1(0) = -\cos\alpha$, $\phi_2(0) = \cos\alpha$ and $\phi_2^1(0) = \sin\alpha$, where k and $\alpha$ are constants. Also show that $W\{\phi_1(x), \phi_2(x)\} = 1$.

**[8 + 8]**

***Q2)*** a) State and prove Sturm's comparison theorem.

b) Define orthogonal functions, orthogonal system and normalised function. Give examples for each

**[10 + 6]**

***Q3)*** a) Find the series solution of $2x^2y^{11} - xy^1 + (x - 5)y = 0$, about the regular singular point of the equation.

b) Obtain generating function for Laguerre polynomials.

c) Express $3x^2 + 2x + 4$ in terms of Laguerre polynomials.

**[6 + 6 + 4]**

***Q4)*** a) Find the general solution of following homogeneous linear equation :

$$\frac{dx}{dt} = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 3 & 1 \\ -3 & -6 & 6 \end{bmatrix} \times$$

b) Define critical point trajectories, saddle point, stable and unstable critical points of a system of differential equations.

**[10 + 6]**

***Q5)*** a) Describe the method to find the nature of critical points, stability and paths of the non-linear system of equations $\frac{dx}{dt} = p(x,y), \frac{dy}{dt} = Q(x,y),$ where $P(x,y)$ and $Q(x,y)$ have continuous partial derivatives.

b) Solve the following Cauchy problem $u_x + u_y + u - 1 = 0$ with the initial data u = sinx on $y = x + x^2$.

**[8 + 8]**

***Q6)*** a) Define complete integral of a non-linear first order partial differential equation. Find the complete integral of the equation $pq - p - q = 0$.

b) Explain Jacobi's method to finding complete integral of partial differential equation. Using the method find the complete integral of $u_x x^2 - u_y^2 - au_z^2 = 0$.

**[8 + 8]**

***Q7)*** a) State Cauchy problem and discuss its geometrical significance.

b) Define semilinear, quasilinear and nonlinear partial differential equations of first order. Explain the method of characteristics for solving quasi linear cauchy problem.

**[8 + 8]**

***Q8)*** a) Solve : $\frac{\partial^3 z}{\partial x^3} + \frac{\partial^3 z}{\partial x^2 \partial y} - \frac{\partial^3 z}{\partial x \partial y^2} - \frac{\partial^3 z}{\partial y^3} = e^{2x+y} + \sin(x+2y).$

b) Discuss the method of characteristics for the Cauchy problem of second order and illustrate with a suitable example.

**[6 + 10]**

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