Sl.No. :

Total No. of Pages : 3

I Year M.Sc. Degree Examination, May - 2026
(SLM Scheme) (Non CBCS) (Second Semester)
MATHEMATICS
Linear Algebra

Time : 3 Hours **Max. Marks : 80**

Instructions : ***1) Answer any five full questions.***
2) All questions carry equal marks

Q1) a) Define a vector space. Explain the general properties of vector space.

b) Express $v = \{1, -2, 3\}$ in $\mathbb{R}^3$ as a linear combination of the following vectors : $v_1 = \{1, 1, 1\}$, $v_2 = \{1, 2, 3\}$, $v_3 = \{2, -1, 1\}$

c) Define a linear transformation. If the mapping $T : R^2 \to R^2$ defined by $T(x, y) = (4x + 5y, 6x - y)$, then show that T is a linear transformation.

[6 + 6 + 4]

Q2) a) State and prove the dimension theorem for linear transformation.

b) Define the rank of a linear transformation. If the matrix of T relative to the basis A and B with matrix $A=[a_{ij}]_{m\times n} = [T]_{BA}$. If u is an arbitrary vector in U. Then prove that $[T(u)]_B - [T]_B [u]_A$.

c) Define algebra of linear transformation and explain its properties.

[6 + 6 + 4]

Q3) a) If V is finite dimensional vector space over a field F, then show that $T \in A(v)$ is non-invertible (Singular) if and only if there exists a $v \neq 0$ in V such that $T(v) = 0$.

b) If A is an algebra, with unit element over F. Then show that A is isomorphic to a sub algebra of A(V) for some vector space V over F.

c) Relative to the basis $B = \{v_1, v_2\} = \{(1, 1), (2,3)\}$ of R^2, find the co-ordinate matrix of

i) $v - (4, -3)$,

ii) $v = (a, b)$

[5 + 6 + 5]

Q4) a) If $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 4 \\ 1 & 4 & 4 \end{bmatrix}$ then find A^{-1}.

b) If $A = [a_{ij}]_n$, then prove that det (A') = det (A).

c) Define minors and cofactors solve the following system of linear equations by cramer's rule

$-2x + 3y - z = 1$

$x + 2y - z = 4$

$-2x - y + z = -3$

[4 + 4 + 8]

Q5) a) Prove that the necessary and sufficient condition for a square matrix A to have O as eigen value is that A is not invertible.

b) If λ is an eigen value of an invertible matrix A over a field F, then show that λ^{-1} is an eigen value of A^{-1}.

c) Let T be a linear operator on a vector space V and let $\lambda_1, \lambda_2, ..., \lambda_k$ be distinct eigen value of T. If $v_1, v_2, ..., v_k$ are eigen vectors of T such that λ_i corresponds to v_i $(1 \leq i \leq k)$, then prove that $\{v_1, v_2, ..., v_k\}$ is linearly independent.

[6 + 4 + 6]

Q6) a) Let $W_1 = (1, 0, 1, 0)$, $W_2 = (1, 1, 1, 1)$ and W3 = (0, 1, 2, 1) in R^4. If $\{w_1, w_2, w_3\}$ is linearly independent. Use the Gram-schmidt Orthogonalization process to compute the orthogonal vectors v_1, v_2 and v_3 and then find the normalize if there vectors to obtain an orthonormal set.

b) Let V be a finite - dimensional inner product space and let T be a linear operator on V. Then show that there exists a unique function $T^* : V \to V$ such that $(T(x),y) = (x, T^*(y))$ for all $x, y \in v$. Furthermore, show that T* is linear.

c) Let B be a bilinear form on a finite dimensional vector space V and let B be an ordered basis for V. Then B is symmetric if and only if ψ_B (B) is symmetric.

[6 + 5 + 5]

Q7) a) Test the matrix $A = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix} \in M_{3\times 3}(R)$ diagonalizable or not.

b) If T and U are simultaneously diagonalizable operators, then show that T and U commute.

c) If $T \in A(V)$ has all its characteristics roots in F, then show that there is a basis of V in which the matrix of T is triangular.

[5 + 5 + 6]

Q8) a) Show that the linear transformation T is diagonalizable if and only if its minimal polynomial is a product of distinct linear factors.

b) Define the rational canonical form. Find all possible rational canonical forms for 6×6 matrices with $(x - 2)(x + 2)^3$ as minimal polynomial.

c) Show that the characteristics polynomial of the companion matrix $c(q(x)$ is $q(x)$.

[4 + 6 + 6]

