MATH-2.1

Sl.No. :

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II Semester M.Sc. Degree Examination, May - 2025
(Non-CBCS, SLM Scheme)
MATHEMATICS
Linear Algebra

Time : 3 Hours **Max. Marks : 80**

Instruction : 1) ***Answer any five full questions.***
2) ***All questions carry equal marks.***

Q1) a) Define a subspace of a vector space. Let R be the field of real numbers and S be the set of all solutions of the equation $x + y + 2z = 0$. Show that S is a subspace of R^3.

b) Prove that the span of any subset S of a vector space V is the subspace of V. Also prove that any subspace of V that contains S must also contain the span of S.

c) Define a linear transformation. Show that product of any two linear transformation from a vector space into itself is also linear.

[5+5+6]

Q2) a) State and prove dimension theorem for linear transformation.

b) Define algebra of linear transformation. Explain the properties of algebra of linear transformation.

c) Relative to the basis $B = \{v_1, v_2\} = \{(1, 1), (2, 3)\}$ of R^2, find the coordinate matrix of $v = (4, -3)$ and $v = (a, b)$.

[8+4+4]

Q3) a) If V is finite dimensional vector space over a field F, then show that $T \in A(V)$ is non-invertible (singular) if and only if there exists a $v \neq 0$ in V such that $T(v) = 0$.

b) If A is an algebra, with unit element. Over F, then show that A is isomorphic to a sub algebra of A(V) for some vector space V over F.

c) Let $T : R^2 \to R^2$ be a linear transformation defined by $T(x, y) = (2x + y, 3x + 2y)$. Then show that T is invertible and find T^{-1}.

[5+6+5]

Q4) a) Describe elementary matrix operations with suitable examples.

b) If a matrix B results from a matrix A by interchanging two rows (columns) of A, then prove that det(B) = – det (A).

c) Define minors and cofactors. Solve the following system of linear equations by cramer's rule

$-2x + 3y - z = 1$

$x + 2y - z = 4$

$-2x - y + z = -3$

[6+4+6]

Q5) a) Prove that the necessary and sufficient condition for a square matrix A to have O as eigen value is that A is not invertible.

b) If λ is an eigen value of an invertible matrix A over a field F, then show that λ^{-1} is an eigen value of A^{-1}.

c) Let T be a linear operator on a vector space V and let $\lambda_1, \lambda_2, ...,\lambda_k$ be distinct eigen values of T. If $V_1, V_2, ...,V_k$ are eigen vectors of T such that λ_1 corresponds to V_1 $(1 \leq i \leq k)$, then prove that $\{V_1, V_2, ...,V_k\}$ is linearly independent.

[6+4+6]

Q6) a) State and prove cauchy schwarz inequality and Triangle inequality.

b) State and prove Gram - Schmidt orthogonalization process.

[8+8]

Q7) a) Show that the linear map satisfying $T^2 = T$ is diagonalizable.

b) If the matrix $A \in F_n$ has all its eigen values in F, then show that there is a matrix $C \in F_n$ such that CAC^{-1} is a triangular matrix.

c) If V is n-dimensional vector space over a field F and $T \in A(V)$ has all its eigen values in F, then show that T satisfies a polynomial of degree *n* over F.

[4+6+6]

Q8) a) Find all possible Jordan forms A for 6×6 matrices with $x^2(1 - x^2)$ as minimal polynomial.

b) If $W \subset V$ is invariant under T, then T induces a linear transformation $\bar{T}$ on V/W, defined by $\bar{T}(v + W) = T(v) + W$. If T satisfies the polynomial $q(x) \in F[x]$, then so does $\bar{T}$. If $p_1(x)$ is the minimal polynomial for $\bar{T}$ over F and if $p_1(x)$ is that for T, then $p_1(x)/p(x)$.

c) Show that the characteristics polynomial of the companion matrix $C(q(x))$ is $q(x)$.

[5+6+5]