Math-2.2

Sl.No. :

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First Year M.Sc. Degree Examination, May - 2026
(SLM Scheme) (Non CBCS) (Second Semester)
MATHEMATICS
Real Analysis - II

Time : 3 Hours **Max. Marks : 80**

Instructions : ***1) Answer any five full questions.***
2) All questions carry equal marks

Q1) a) Let $A \subseteq R$ and $f : A \to \mathbb{R}$ then show that f is continuous at the point $x_0 \in A$, $\lim_{x \to x_0} f = L$ if and only if for every sequence $\{x_n\}$ in A with $x_n \neq x_0 \ \forall n$, $\lim_{n \to \infty} x_n = x_0 \Rightarrow \lim_{n \to \infty} f(x_n) = L.f(x)$.

b) Prove that the function f on $\mathbb{R}$ is continuous on R if and only if for every open set G in R, $f^{-1}(G)$ is open in R.

[8 + 8]

Q2) a) Show that continuous image of a compact set is compact.

b) Define a uniform continuous function varity continuity for the function $f(x) = \frac{1}{x}$ on $A = (0,1)$.

c) Show that $\lim_{x \to a} \frac{1}{x} = \frac{1}{a}$ if $a > 0$.

[6 + 5 + 5]

Q3) a) Let $f : [a, b] \to \mathbb{R}$ and if f has a local maximum at a point $x_0 \in (a, b)$ and if $f'(x_0)$ exists then prove that $f'(x_0) = 0$.

b) If f is a differentiable function on [a, b] and λ is a number with $f'(a) < \lambda < f'(b)$ then show that there exists at least one $C \in (a,b)$ such that $f'(c) = \lambda$.

c) Show that $f(x) = x^2 \in \mathbb{R}[x^2]$ on [0, 1]

[6 + 6 + 4]

Q4) a) If p* be the refinement of P prove that

i) $L(p, f, \alpha) \leq L(p^*, f, \alpha)$

ii) $U(p, f, \alpha) \geq U(p^*, f, \alpha)$

iii) $\underline{\int_a^b} f\, d\alpha \leq \int_a^{b-} f\, d\alpha$

b) Evaluate $\int_0^3 x\, d([x])$, where $[x]$ is the gretest integer function.

[8 + 8]

Q5) a) If $f \in R(\alpha)$ on [a, b] and if $|f(x)| < M$ for all $x \in [a, b]$, then show that $\left|\int_a^b f\, d\alpha\right| \leq M(\alpha(b) - \alpha(a))$.

b) State and prove first mean value theorem for Reimann - stieltje's integral.

[8 + 8]

Q6) a) Let f be Riemann integrable function on [a, b]. If there is a differentiable function g on [a, b] such that $g' = f$ then prove that $\int_a^b f(x)dx = g(b) - g(a)$.

b) Define a function of bounded variation show that a monotonic and bounded function f on $[a, b]$ is a function of bounded variation on $[a, b]$ and $v(f) = |f(b) - f(a)|$.

[8 + 8]

Q7) a) State and prove the theorem on uniform convergence and integration.

b) If $\{f_n\}$ is a squence of continuous functions on $A \subseteq \mathbb{R}$ and $f_n \to f$ uniformly on A. Then prove that f is continuous on A.

[10 + 6]

Q8) a) State and prove stone - weierstrass theorem.

b) Define an expenential function $E(x)$. Prove that $E(x)$ satisfies the following properties :

i) $E(x)\, E(y) = E(x + y)$

ii) $E(q) = e^q$, for every rational number q, where $e = \sum_{n=0}^{\infty} \frac{1}{n!}$.

iii) $E(x)E(-x) = E(x-x) = E(0)=1$

iv) $E'(x) = Ex$.

[8 + 8]

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