Math-2.2

Sl.No. :

Total No. of Pages : 3

II Semester M.Sc. Degree Examination, May - 2025

(Non CBCS, SLM Scheme)

MATHEMATICS

Real Analysis - II

Time : 3 Hours **Max. Marks : 80**

Instruction: ***Answer any five full questions:***

Q1) a) Define limit of a function. Let $A \subseteq \mathbb{R}$. Let f and g be function from A to $\mathbb{R}$. If $x_0 \in \mathbb{R}$ then prove the following.

i) $\lim_{x \to x_0}(fg)x = \left(\lim_{x \to x_0} f(x)\right)\left(\lim_{x \to x_0} g(x)\right)$

ii) $\lim_{x \to x_0}(af)(x) = a \lim_{x \to x_0} f(x)$

b) State and prove sequential criterion of continuity of a function.

[8+8]

Q2) a) Let $A \subseteq \mathbb{R}$ and let $f : \mathbb{R} \to \mathbb{R}$. Then prove that function f is continuous on $\mathbb{R}$ if and only if for easy open set G in $\mathbb{R}$, $f^{-1}(G)$ is open in $\mathbb{R}$.

b) Define uniform continuity. Show that the function $f(x) = x^2$ $\forall x \in \mathbb{R}^1$ and A = (0,b) where b > 0. The function is uniformly continuous on A.

[8+8]

Q3) a) Let E be a non compact set in $\mathbb{R}$. Then prove the following.

i) There exists a continuous function on E which is not bounded.

ii) There exists a continuous and bounded function on E which has no maximum.

iii) There exists a continuous function which is not uniformly continuous.

b) Define connected set. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous map. If E is a connected subset of $\mathbb{R}$ then prove that f(E) is also connected.

[10+6]

Q4) a) If f is a monotone function on an open interval (a, b) then prove that any discontinuity that f may have in this interval is of the first kind.

b) Show that the function $f(x) = \begin{cases} x^2 \sin 1/x, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$ is differentiable but not continuous.

[10+6]

Q5) a) State and prove Generalized mean value theorem.

b) Find $\lim_{x \to 0} \frac{1 - \cos x}{x^2}$.

c) State and prove Taylor's theorem.

[4+4+8]

Q6) a) Let $[a, b]$ be the given internal and let p and p^* be partitions of $[a, b]$ such that p^* is a refinement p of then prove

i) $L(p, f, \alpha) \leq L(p^*, f, \alpha)$

ii) $U(p, f, \alpha) \geq U(p^*, f, \alpha)$

where f is a bounded function and α is a monotonically increasing function on $[a, b]$.

b) Evaluate $\int_0^3 x \, d([x])$, where $[x]$ is the greatest integer function.

[8+8]

Q7) a) If $f_1 \in \mathbb{R}(\alpha)$ and $f_2 \in \mathbb{R}(\alpha)$ on $[a, b]$ then $f_1 + f_2 \in \mathbb{R}(\alpha)$ on $[a, b]$ and show that $\int_a^b (f_1 + f_2)\,d\alpha = \int_a^b f_1\,d\alpha + \int_a^b f_2\,d\alpha$.

b) Let $f \in \mathbb{R}(\alpha)$ on $[a,b]$ and let $g(x) = \int_a^x f(t)\,dt \quad \forall x \in [a,t]$. Then $g(x)$ is continuous on $[a, b]$. Further if f is continuous at $x_0 \in [a,b]$ then g is differentiable at x_0 and show that $g'(x_0) = f(x_0)$.

[8+8]

Q8) a) State and prove Cauchy criterion for uniform convergence.

b) State and prove Stone Weierstrass theorem.

[8+8]

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