Math-2.3

Sl.No. :

Total No. of Pages : 2

II Semester M.Sc. Degree Examination, June - 2023
(Non CBCS, SLM Scheme)
MATHEMATICS
Complex Analysis-II

Time : 3 Hours **Max. Marks : 80**

Instructions : ***1) Answer any five full questions.***
2) All questions carry equal marks.

Q1) a) State and prove argument principle.

b) Show that $f(z) = z^5 + 5z^3 + z - 2$ has three of its roots in the disk $|z|<1$ and all its roots in the disk $|z|<\frac{5}{2}$.

c) Find the residue of the function $f(z)=\frac{z^4+1}{z(z^2-1)}$ at $z = 0$.

[8+4+4]

Q2) a) If $\lim_{z\to\infty} f(z)=0$ and $f(z)$ is meromorphic in upper half plane then prove that $\lim_{R\to\infty}\int_{\gamma} e^{icz}\ f(z)\ dz=0,\quad (c>0)$ where γ denote the semi-circle $|z|=R$, $\text{Img}(z) > 0$.

b) Evaluate the following integral by residue method

i) $\int_0^{\infty}\frac{\sin mx}{x(x^2+a^2)^2}\,.\,dx\ ;\ m>0,\ a>0$

ii) $\int_{-\infty}^{\infty}\frac{\log(1+x^2)}{1+x^2}.\,dx$

[6+10]

Q3) a) If u_1 and u_2 are harmonic in a region D, then prove that $\int_\gamma u_1 *du_2 - u_2 *du_1 = 0$ for every cycle γ which is homologous to zero in D.

b) Define Poisson kernel. If $r > 0$ then prove that $\frac{1}{2\pi}\int_0^{2\pi} P(z, re^{i\theta})\, d\theta = 1.$

c) Discuss the convergence of the geometric series $\sum_{n=0}^{\infty} z^n$.

[6+6+4]

Q4) a) State and prove Schwarz's theorem.

b) State and prove Weierstrass theorem.

[8+8]

Q5) a) Show that the formula $f(z) = \sum_{n=1}^{\infty} \frac{1}{n^z}$ defines an analytic function in the open set U={z : Re z > 1}. Find a series expansion for f' that is valid in U.

b) State and prove Laurent's theorem.

[6+10]

Q6) a) Prove that an infinite product $\prod (1+a_n)$ is convergent if and only if the series $\sum \log(1+a_n)$ is convergent, the principle value of the logarithm being taken in each case.

b) Represent $\sin \pi z$ in the form of canonical product.

[8+8]

Q7) a) Derive Legendre duplication formula for Gamma function.

b) State and prove Jensen's formula.

[8+8]

Q8) a) For x = Re(z) > 1, prove that $\frac{1}{\xi(z)} = \Pi(1 - p_n^{-5})$ where the product is taken over all prime number p.

b) Define Riemann's zeta function $\zeta(z)$. Prove that the zeta function $\zeta(z)$ is a meromorphic function on the complex plane with only one pole, a simple pole at z =1 with residue 1.

[8+8]

❖ ❖ ❖