Sl.No. :

Total No. of Pages : 3

First Year M.Sc. Degree Examination, May - 2026
(SLM Scheme) (NON CBCS) (II Semester)
MATHEMATICS
Math 2.3 : complex Analysis - II

Time : 3 Hours **Max. Marks : 80**

Instruction : Answer any five full questions. All questions carry Equal marks.

Q1) a) State and Prove Arguement Principle.

b) State and Prove Cauchy's Residue theorem.

c) How many roots does the Equation $z^4 - 6z + 3 = 0$ have in the disk $|z| < 2$?

[7+7+2]

Q2) a) If $f(z)$ is a meromorphic function in the upper half plane and $f(z) \to 0$ uniformly as $z \to \infty$ then prove that $\lim_{R\to\infty} \int_{\gamma} e^{icz} f(z)\,dz = 0, (c>0)$ where γ is the semicircle $|z| = R$, $\text{Im}(z) > 0$

b) Evaluate $\int_0^{\pi} \sin^{2n}\theta\, d\theta$ for 'n' a positive integer

c) Evaluate $\int_{|z|=1} \frac{(2z+1)^2}{\tan^{-1}\left(\frac{ix}{2}\right)} dz$.

[6+7+3]

Math 2.3

Q3) a) State and Prove the mean value property for harmonic functions.

b) State and Prove Poisson integral formula.

c) Derive Polar form of the Poisson Kernel.

[7+7+2]

Q4) a) Define uniform convergence of a Sequence of functions. Suppose $\{f_n\}$ converges uniformly to a function f on a set S. If each f_n is continuous on S then prove that f is continuous on S. Furthermore, prove that $\int_\gamma f(z)dz = \lim_{n\to\infty}\int_\gamma f_n(z)dz$ for every piecewise smooth path γ in S.

b) State Taylor's theorem. Expand $f(z) = \dfrac{z-1}{z+1}$ as a Taylor series.

i) about $z = 0$ and

ii) about $z = 1$

[8+8]

Q5) a) Prove that the infinite produce $\prod_{n=1}^{\infty}(1+a_n)$ is convergent if and only if the series $\sum_{n=1}^{\infty}\log(1+a_n)$ is convergent, where the principal value of the logarithm being taken in Each case.

b) Represent $\sin\pi z$ in the form of canonical product.

[8+8]

Q6) a) State and Prove legendre's duplication formula Find the value of $\Gamma\left(\dfrac{1}{2}\right)$

b) Prove that $\sqrt{\pi}\ \Gamma(2z) = 2^{2n-1}\ \Gamma(z)\Gamma\left(z+\dfrac{1}{2}\right)$

[8+8]

Math 2.3

Q7) a) State and Prove Poisson-Jonsen formula.

b) State and Prove Hadamard's three circle theorem.

[8+8]

Q8) a) For $x = \mathrm{Re}(z) > 1$, then prove that

$$\frac{1}{\xi(z)} = \prod_{p}\left(1 - p_n^{-3}\right)$$ where the product is taken over all prime numbers p.

b) Prove that the function $\xi(z)$ satisfies the functional Equation

$$\xi(z) = 2^z \Pi^{z-1} \sin\left(\frac{\Pi z}{2}\right)\Gamma(1-z)\xi(1-z)$$

[8+8]