Sl.No. :

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II Semester M.Sc. Degree Examination, May-2025
(Non-CBCS, SLM Scheme)
MATHEMATICS
Complex Analysis - II

Time : 3 Hours **Max. Marks : 80**

Instruction : Answer any five full questions. All questions carry equal marks.

Q1) a) State and Prove the Rouche's theorem and hence prove that the fundamental theorem of Algebra.

b) If a function f has an isolated singularity at ∞, then prove that Res $(f, \infty) =$ –Res $[z^{-2} f(z^{-1}), 0]$ Corroborate this fact.

c) Define residue of the function $f(z)$ at an isolated Singularity.

[8+6+2]

Q2) a) Evaluate the following integrals

i) $\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)(x^2+4)}$.

ii) $\int_{0}^{\infty} \frac{\sin cx}{x} dx,\ c > 0$

b) If $\lim_{z \to a} (z-a) f(z)$ A and if C, is the arc $\theta_1 \le \theta \le \theta_2$ of the circle $|z-a| = r$, then prove that $\lim_{r \to 0} \int_c f(z)\,dz = i\,\mathrm{A}\,(\theta_2 - \theta_1)$.

[8+8]

Q3) a) State and prove Schwarz theorem.

b) State and prove reflection principle.

[8+8]

Q4) a) State and Prove the weierstrass theorem for a sequence analytic function.

b) State and Prove the weierstrass M - test.

c) Define Pointwise Convergence of a sequence of function. Give one example.

[8+4+4]

Q5) a) State and Prove the Laurent's theorem.

b) Prove that $\lim_{n\to\infty}\left[1+\frac{1}{2}+\frac{1}{3}+..+\frac{1}{n}-\log n\right]$ Exists.

[10+6]

Q6) a) Prove that $\Gamma(z)=\lim_{z\to\infty}\frac{n!n^z}{z(z+1)(z+2)...(z+n)}$.

b) Prove that $\Gamma(z)=\int_0^\infty e^{-t}t^{z-1}dz,\ \mathrm{Re}(z)>0$

c) Define Euler's Constant *r*, prove that it is a Constant.

[4+6+6]

Q7) a) If the real part of an Entire function $g(z)$ satisfies the inequality $\mathrm{Re}\{g(z)\}<r^{p+\varepsilon}$ for Every $\varepsilon>0$ and all sufficiently large *r*, then prove that $g(z)$ is a polynomial of degree not Exceeding *p*.

b) If $f(z)$ is an Entire function of order *p*, then prove that for Every $\varepsilon>0$ the inequality $\mathrm{N}(r)\le r^{p+\varepsilon}$ holds for all sufficiently large r, where N (*r*) denote the number of zeros of an Entire function $f(z)$ in the closed disk $|z|\le r$.

[9+7]

Q8) a) Let$(-s)^{z-1}$ be defined on the complement of the positive real axis as $e^{(z-1)\log(-s)}$ with $-\pi<\mathrm{Im}\log(-s)<\pi$ and let $\mathrm{Re}(z)>1$, then prove that

$$\xi(z)=\frac{-\Gamma(1-z)}{2\pi i}\int_r\frac{(-s)^{Z-1}}{e^s-1}ds,$$

Where *r* is the contour made up of two infinite paths C and C_n, both beginning and ending near the positive real axis.

b) Let *f* be a non-constant analytic function and let

$$P_1=\inf\{\lambda\ge 0: m(r)\le e^{r\lambda}\ \text{for sufficiently large}\ r\}$$

$$P_2=\limsup_{r\to\infty}\left\{\frac{\log\log m(r)}{\log r}\right\}.$$ then prove that $P_1=P_2$

c) Prove that zeta function $\xi(z)$ is a meromorphic function on the complex plane with only one pole at $z=1$ with residue 1.

[8+4+4]

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