Math-2.4

Sl.No. :

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II Semester M.Sc. Degree Examination, June - 2023
(Non-CBCS, SLM Scheme)
MATHEMATICS
Numerical Analysis

Time : 3 Hours **Max. Marks : 80**

Instructions: ***1) Answer any five questions.***
2) All questions carry equal marks.

Q1) a) Derive the Regula - Falsi method to find the root of $f(x) = 0$. Using the method find the smallest positive root of $x^3 - 5x + 1 = 0$ perform 3 iterations. **[8]**

b) Using the Muller method find the root of the equation $\cos x - xe^x = 0$, correct to 5 decimal places. Use the suitable initial approximations $x_0 = 0, x_1 = 0.1$ and $x_2 = 0.2$. **[8]**

Q2) a) Define the rate of convergence of an iterative method. Show that the Secant method has rate of convergence 1.618. **[5]**

b) Show that the iterative method given by the sequence $x_{n+1} = \frac{1}{2} x_n \left(1 + \frac{a}{x_n^2}\right)$ converges to the limit $\sqrt{a}$ and the rate of convergence is 2. **[5]**

c) Perform two iterations of the linear iteration method followed by one iteration of the Aitken Δ^2 method to find the root of the equation. **[6]**
$f(x) = x^3 - 5x + 1 = 0, x_0 = 0.5$.

Q3) a) Obtain the Strum's Sequence for the polynomial $4x^4 + 2x^2 - 1$. Using the Strum's sequence find the number of real and complex roots. **[8]**

b) Perform two iterations of Bairstow method to extract a quadratic factor $x^2 + px + q$ from the polynomial $x^4 + x^3 + 2x^2 + x + 1 = 0$. Use the initial approximations $x^2 + p_0 = 0.5$, and $q_0 = 0.5$. **[8]**

Q4) a) Derive the Newton's Divided Difference interpolating polynomial with the truncation error. **[6]**

b) The following table of values is given. Obtain the Newton's Backward interpolating polynomial and hence find the approximate value of $f(4.5)$. **[6]**

x	2	3	4	5
$f(x)$	16	81	256	625

c) With usual notations prove the following. **[4]**

i) $\delta = \nabla(1-\nabla)^{-1/2}$

ii) $\nabla - \Delta = -\Delta\nabla$

Q5) a) Derive the Newton's bivariate interpolating polynomial for equi-spaced points. **[8]**

b) Obtain the Least Square Polynomial approximation of first degree and second degree to the function $f(x) = x^3$ on $[0, 1]$. **[8]**

Q6) a) The following table of values is given : **[8]**

x	–1	1	2	3	4	5	7
$f(x)$	1	1	16	81	256	625	2401

Using the formula $f'(x_1) = (f(x_2) - f(x_0))/(2h)$ and the Richardson extrapolation, find $f'(3)$.

b) Describe the Simpson's rule to find the approximate value of the definite integrals. Evaluate the integral $I = \int_0^1 \frac{dx}{1+x}$ composite Simpson's rule, with 4 and 8 equal subintervals. **[8]**

Q7) a) Explain stability analysis of Single - step methods using the Runge - Kutta fourth method. **[8]**

b) Derive Taylor series method to solve $u' = f(t, u)$, $u(t_0) = \eta_0$, $t \in [t_0, b]$. Using the method solve the initial value problem $u' = 2t - u$, $u(0) = -1$. **[8]**

Q8) a) Discuss the Stability of PM_pCM_c method for the solution of the initial value problems. **[8]**

b) Explain linear method to solve second order differential equations using boundary conditions. **[8]**

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