MATH-3.1

Sl.No. :

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III Semester M.Sc. Degree Examination, May - 2023
(Non-CBCS, SLM Scheme)
MATHEMATICS
Topology

Time : 3 Hours **Max. Marks : 80**

Instructions: ***1) Answer any five full questions.***
2) All questions carry equal marks.

Q1) a) Let $(y,\tau^*) \subseteq (x,\tau)$ and $E \subset y \subset x$, then the following holds.

i) $\overline{E}\, Y = \overline{E} \cap Y$

ii) $E^o = E_y^{\circ} \cap y^o$

iii) $b_y(E) = b(E) \cap y$

b) Prove that a bijective mapping $f : x \to y$ is a homeomorphism if and only if $f(\overline{A}) = \overline{f(A)}, \forall A \subseteq X$.

[9+7]

Q2) a) Prove that a topological space (x,τ) is connected if and only if the only continuous mapping from X to the 2-Point space over the constant mappings.

b) Prove that a compact subset of a Hansdorff space is closed.

[10+6]

Q3) a) Given a mapping $f : x \to y$. Then prove that the following are equivalent.

i) f is continuous

ii) B is closed in $y \Rightarrow f^{-1}(B)$ is closed in *x*.

iii) $f(\overline{A}) \subseteq \overline{f(A)}, \forall A \subseteq X$

b) Prove that a point *x* belongs to the closure of a set A if and only if every open set G which contains *x* has a non-empty intersection with A.

[12+4]

Q4) a) If every countable open cover of (x,τ) has a finite subcover then prove that X is countably compact.

b) Prove that a patch-connected subset is a connected subset.

[8+8]

Q5) a) Prove that a topological space (x,τ) is a T_1- space if and only if all singleton sets are closed.

b) Prove that a compact Hansdorff space is normal.

[8+8]

Q6) State and prove Urysohn's Metrization theorem. **[16]**

Q7) State and prove stones theorem. **[16]**

Q8) a) Prove that every paracompact regular space is normal.

b) If X is a regular paracompact space and Y is a regular $\sigma-$compact space. Then prove that X×Y is paracompact.

[8+8]

