Math 3.1

Sl.No. :

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Second Year M.Sc. Degree Examination, May - 2026
(SLM Scheme) (NON CBCS) (Third Semester)
MATHEMATICS
Math 3.1 : Topology

Time : 3 Hours **Max. Marks : 80**

Instruction : Answer any five full questions.

Q1) a) Define a topology on a non-empty set. Prove the following hold in (X, τ)

i) $d\phi = \phi$

ii) $A \subseteq B \Rightarrow d(A) \subseteq d(B)$

iii) $d(A \cup B) = d(A) \cup d(B)$

b) Let (X. τ) be a topological space. Prove that a sub-family β of τ is a base for τ if and only if $U \in \tau$ and $x \in U$ implies there is a B in β such that $x \in B \subseteq U$.

[8+8]

Q2) a) Show that $f : x \to y$ is continuous iff $\overline{f^{-1}(B)} \subseteq f^{-1}(\overline{B})$, for all $B \subseteq Y$.

b) Define a metric space. Let X be a metric space with metric d. Define $\overline{d}(x, y) = \min \{d(x, y), 1\}$ Then show that $\overline{d}$ is a metric that induces the topology of X.

[6+10]

Q3) a) If C is a connected subset of (X. τ) and $C \subseteq Y \subseteq \overline{C}$ then prove that y is connected.

b) Prove that continuous image of a connected space is connected.

c) If $\{A_i\}$ is the family of connected subsets of X such that $\bigcap A_i \neq \phi$ then prove that $\bigcup_i A_i$ is connected

[5+5+6]

Q4) a) Prove that a separable metric space is second countable.

b) Show that compact matric space is complete.

[8+8]

Q5) a) Define separation axioms. Show that in a T_0 - space, the closures of distinct points are distinct and conversely.

b) Prove that a compact Hausdorff space is normal.

[8+8]

Q6) State and prove Urysohn's Metrization Theorem. **[16]**

Q7) a) Let $\{U_1, ..., U_n\}$ be a finite open covering of the normal space X, Then prove that there exists a partition of unity dominates by $\{U_i\}$

b) Prove that a compact subset of a Hausdorff space is closed.

[8+8]

Q8) a) Prove that a Hausdorff space has a smallest compactification if and only if it is locally compact.

b) Prove that every para compact space is normal

[8+8]