Math-3.1

Sl.No. :

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III Semester M.Sc. Degree Examination, May - 2025

(Non - CBCS, SLM Scheme)

MATHEMATICS

Topology

Time : 3 Hours **Max. Marks : 80**

Answer any five full questions:

Q1) a) Define the following terms:

i) Topological space

ii) Relative topology

iii) Base for the topology

iv) Product topology

b) If Z is a subspace of Y and Y is a subspace of X then prove that Z is a subspace of X.

c) Define closure of a set. Prove that a point x belongs to the closure of a set A if and only if every open set G containing x has a non-empty intersection with A.

[4+6+6]

Q2) a) Prove that a mapping $f:(X,\tau)\to(Y,U)$ is continuous at x belongs to X if and only if v is a neighbourhood of $f(x)$ implies $f^{-1}(v)$ is a neighbourhood of x.

b) Define a metric space. Let X be a metric space with metric d. Define $d^{-}(x, y) = \min\{d(x, y),1\}$ then show that d^{-} is a metric that induces the topology of X.

[6+10]

Q3) a) Prove that the following statements are equivalent.

i) X is connected

ii) X and ϕ are the only sets which are both open and closed.

iii) X is not the union of two non-empty disjoint open sets.

iv) X is not the union of two non-empty disjoint closed sets.

b) If L is a linear continuum in the order topology then show that L is connected and so is every interval and ray in L.

[8+8]

Q4) a) Prove that (X,τ) is compact if and only if every family of closed sets having finite intersection property has a non-empty intersection.

b) Show that every finite set is sequentially compact.

[10+6]

Q5) a) State and prove Tietze's extension theorem.

b) Prove that A normal space is regular iff it is completely regular.

[12+4]

Q6) a) Prove that a countably compact metric space is totally bounded.

b) Define second countable space and prove that every second countable space is first countable space.

c) Prove that a compact Hausderff space is normal.

[4+4+8]

Q7) State and prove Tychonoff theorem. **[16]**

Q8) a) If A be a locally finite collection of subsets of a topogical space X. Then prove that $\overline{\bigcup_{A\in A} A} = \bigcup_{A\in A} \overline{A}$

b) Prove that every Para compact regular space is normal.

[8+8]

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