Math-3.2

Sl.No. :

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III Semester M.Sc. Degree Examination, May - 2023
(Non-CBCS, SLM Scheme)
MATHEMATICS
Measure and Integration

Time : 3 Hours **Max. Marks : 80**

Instruction: 1) ***Answer any five questions.***

2) ***All questions carry equal marks.***

Q1) a) Let A be an algebra of X and $\{A_i\}$ a sequence of sets in A. Then prove that we can find a sequence $\{B_j\}$ of sets in A, such that $B_n \cap B_m = \phi$ for $n \neq m$ and $\bigcup_{i=1}^{\infty} B_j = \bigcup_{i=1}^{\infty} A_i$. **[8]**

b) Let $\{A_n\}$ be a countable collection of sets of real numbers. Then show that $m^*\left(\bigcup_{n=1}^{\infty} A_n\right) \leq \sum_{n=1}^{\infty} m^* A$. **[8]**

Q2) a) State and prove Littlewood's three principles. **[8]**

b) If c is a constant and the functions f and g are measurable real-valued functions defined on the same domain, then show that the functions $f + c$, cf, $f + g$ and $f - g$ are measurable. **[8]**

Q3) a) If f is a bounded function defined on a measurable set E with $mE < \infty$. Then prove that f is measurable if and only if $\inf_{\psi \geq f} \int \psi = \sup_{\phi \leq f} \int \phi$. **[10]**

b) State and prove Bounded Convergence theorem. **[6]**

Q4) a) If f and g are non-negative measurable functions defined on a measurable set E, then prove the following. **[10]**

i) $\int_E cf = c\int_E f,\ c > 0$

ii) $\int_E (f+g) = \int_E f + \int_E g$

iii) If $f \le g$ almost everywhere, then $\int_E f \le \int_E g$

b) State and prove Lebesgue convergence theorem for integrable functions. **[6]**

Q5) a) Prove that a function f is of bounded variation over $[a, b]$ if and only if f is the difference of two monotone real-valued functions on $[a, b]$. **[8]**

b) If f is of bounded variation on $[a, b]$, then prove the following : **[8]**

i) P – N = $f(b) - f(a)$

ii) P + N = T

Q6) a) If f is bounded and measurable on $[a, b]$ and $\mathrm{F}(x) = \int_a^x f(t)\,dt + \mathrm{F}(a)$, then prove that $\mathrm{F}'(x) = f(x)$ almost everywhere in $[a, b]$. **[8]**

b) If f is absolutely continuous on $[a, b]$ and $f'(x) = 0$ almost everywhere then prove that f is constant. **[8]**

Q7) a) Let $(X, \mathcal{B}, \mu)$ be a measure space. If $E_i \in \mathcal{B}$ for $i = 1, 2, 3,.....$, $\mu E_1 < \infty$ and $E_i \supset E_{i+1}$, $i = 1, 2, 3,.....$ then prove that $\mu\left(\bigcap_{i=1}^{\infty} E_i\right) = \lim_{n\to\infty} \mu E_n$. **[8]**

b) Let $\{f_n\}$ be a sequence of non-negative measurable functions. Then prove that $\int \sum_{n=1}^{\infty} f_n = \sum_{n=1}^{\infty} \int f_n$. **[8]**

Q8) a) State and prove Lebesgue Decomposition theorem. **[8]**

b) Let $\{E_i\}$ be a disjoint sequence of measurable sets and let A be any set, then prove that $\mu^*(A \cap E) = \sum_{i=1}^{\infty} (A \cap E_i)$ where $E = \bigcup_{i=1}^{\infty} E_i$. **[8]**

