Sl.No. :

Total No. of Pages : 2

III Semester M.Sc. Degree Examination, May - 2026
(Scheme : Non - CBCS)
MATHEMATICS
Measure and Integration

Time : 3 Hours **Max. Marks : 80**

Instruction : ***Answer any five full questions.***

Q1) a) Define lebesgue outer measure and prove that the lebesgue outer measure of an interval is its length.

b) Let A be an algebra of X and $\{A_i\}$ is a sequence of sets in A. Then prove that we can find a sequence $\{B_i\}$ of sets in A such that $B_n \cap B_m = \phi$ for $n \neq m$ and $\bigcup_{i=1}^{\infty} B_i = \bigcup_{i=1}^{\infty} A_i$.

[8+8]

Q2) a) Show that the collection M of Lebesgue measurable sets is a σ-algebra.

b) Let $\{E_n\}$ be an infinite decreasing sequence of Lebesgue measurable sets that is a sequence with $E_{n+1} \subset E_n$, $n = 1, 2, 3, \ldots$. Let mE_1 be finite then prove that $m\left(\bigcap_{i=1}^{\infty} E_i\right) = \lim_{n\to\infty} mE_n$.

[8+8]

Q3) a) If c is a constant and the function f and g are measurable real-valued functions defined on the same domain, then show that the functions $f + c$, cf, $f + g$, $g - f$ and fg are also measurable.

b) If ϕ and ψ are simple functions which vanish outside a set of finite measure then prove that $\int(a\phi + b\psi) = a\int\phi + b\int\psi$ and if $\phi \geq \psi$ then $\int\phi \geq \int\psi$.

[8+8]

Q4) a) If f and g are non negative measurable functions defined on a measurable set E then prove that

i) $\int_E cf = c\int_E f, \qquad c > 0$

ii) $\int_E (f+g) = \int_E f + \int_E g$

iii) If $f \le g$ a.e., then $\int_E f \le \int_E g$

b) State and prove Lebesgue convergence theorem for integrable functions.

[8+8]

Q5) a) Prove that the function f is of bounded variation over $[a, b]$ if and only if f is the difference of two monotone real valued functions on $[a, b]$.

b) State and prove Vitali Lemma.

[4+12]

Q6) a) If f is integrable on $[a, b]$ and $\int_a^x f(t)dt = 0 \ \ \forall x \in [a,b]$ then prove that $f(t) = 0$ a.e., on $[a, b]$.

b) If f is absolutely continuous on $[a, b]$ then show that f has a derivative almost everywhere.

[8+8]

Q7) a) Let (x, B, μ) be a measure space. If $E_i \in B$ for $i = 1, 2, 3, ...$, $\mu E_i < \infty$ and $E_i \ E_{i+1}$, $i = 1, 2, 3,....$, then P.T. $\mu\left(\bigcap_{i=1}^{\infty} E_i\right) = \lim_{n\to\infty} \mu E_n$.

b) State and prove Fatou's Lemma.

[8+8]

Q8) a) State and prove Lebesgue convergence theorem.

b) Let $\{E_i\}$ be a disjoint sequence of measurable set and Let A be any set then prove that $\mu^*(A \cap E) = \sum_{i=1}^{\infty}(A \cap E_i)$, where $E = \bigcup_{i=1}^{\infty} E_i$.

[8+8]

