Math-3.2

Sl.No. :

Total No. of Pages : 2

III Semester M.Sc. Degree Examination, May - 2025

(Non-CBCS, SLM Scheme)

MATHEMATICS

Measure and Integration

Time : 3 Hours **Max. Marks : 80**

Instructions: ***1) Answer any five full questions.***

2) All questions carry equal marks.

Q1) a) Let A be an algebra of X and $\{A_i\}$ a sequence of sets in A. Then prove that there is a sequence $\{B_i\}$ of sets in A such that $B_n \cap B_m = \phi$ for $n \neq m$ and $\bigcup_{i=1}^{\infty} B_i = \bigcup_{i=1}^{\infty} A_i$

b) Prove that the interval (a,∞) is Lebesgue measurable.

[8+8]

Q2) a) Let $\{E_n\}$ be an infinite decreasing sequence of Lebesgue measurable sets, that is a sequence with $E_{n+1} \subset E_n, n = 1,2,3,.....$ Let mE_1 be finite. Then prove that $m\left(\bigcap_{i=1}^{\infty} E_i\right) = \lim_{n\to\infty} mE_n$.

b) If C is a constant and the functions f and g are measurable real-valued functions defined on the same domain, then show that the functions $f + C$, Cf, $f+g$ and $g-f$ are measurable

[8+8]

Q3) a) If f is a bounded function defined on a measurable set E with mE< ∞. Then prove that f is measurable if and only if, $\inf \int_{\psi \geq f} \psi = \sup \int_{\phi \leq f} \phi$.

b) State and prove bounded convergence theorem for a measurable function.

[8+8]

Q4) a) State and prove Fatou's lemma for a non-negative measurable function.

b) Let f and g be non-negative measurable functions. If f is integrable over E and $g(x) < f(x)$ on E then prove that g is also integrable on E and $\int (f-g) = \int f - \int g$.

c) State and prove Lebesgue convergence theorem for integrable functions.

[6+4+6]

Q5) a) Prove that a function f is of bounded variation over [a, b] if and only if f is the difference of two monotone real-valued functions on [a,b].

b) If f is bounded and measurable on [a,b] and $F(x) = \int_a^x f(t)\,dt + F(a)$, then prove that F'($x$) = $f(x)$ almost everywhere in [a,b].

[8+8]

Q6) a) Prove that a continuous function need not be of bounded variation.

b) Let $\{f_n\}$ be a sequence of non-negative measurable functions. Then prove that $\int \sum_{n=1}^{\infty} f_n = \sum_{n=1}^{\infty} \int f_n$.

[8+8]

Q7) a) State and prove Hahn Decomposition theorem.

b) State and prove Labesgue Decomposition theorem.

[8+8]

Q8) State and prove Radon - Nikodym theorem. **[16]**

❋ ❋ ❋