Math-3.3

Sl.No. :

Total No. of Pages : 2

III Semester M.Sc. Degree Examination, May - 2025

(Non - CBCS, SLM Scheme)

MATHEMATICS

Functional Analysis

Time : 3 Hours **Max. Marks : 80**

Instruction: ***Answer any five questions:***

Q1) a) Define an open ball and open set in a metric space. Show that a set G is an open set in a metric space (X, d) if and only if G is a union of open spheres.

b) Let (X,d) be a complete metric space and (X,d) be a subspace of (X,d). Then show that Y is complete if and only if Y is closed.

[8+8]

Q2) State and prove Picard's theorem. **[16]**

Q3) a) Prove that a complete and totally bounded metric space is compact.

b) State and prove Ascoli-Arzela theorem.

[8+8]

Q4) a) Define a normed spaces. Show that a normed linear space X is complete if and only if every absolutely convergent series in X is convergent.

b) State and prove Hahn-Banach theorem for Normed linear Space.

[8+8]

Q5) a) Prove that the normed linear space B (N, N') is a Banach space whenever N' is a Banach space.

b) Let M be a closed subspace of a normed Linear space N and Let $x_0 \notin M$. Then $\exists\ f_0 \in N^*$, such that $f_0(M) = \{0\}$ and $f_0(x_0) \neq 0$.

[8+8]

Q6) a) Show that with the example every continuous linear aperator is also a closed linear operator but not converse true.

b) Let N be a normed linear space. Show that a nonempty subset X of N is bounded if and only if $f(x)$ is a bounded set of members for every $f \in N^*$.

[8+8]

Q7) a) State and prove uniform Bounded principle.

b) Prove the following two operations of a Hilbert space it :

i) $||x + y||^2 + || x–y ||^2 = 2 (||x||^2 + ||y||^2)$

ii) $4 (x,y) = ||x+y||^2 – ||x–y||^2 + i\ ||x + i\ y||^2 –i||\ x–iy||^2$

[8+8]

Q8) a) Let M be a closed Linear subspace of a Hilbert space H. Then prove that $H = M \oplus M^{\perp}$

b) State and prove Riesz representation theorem.

[8+8]

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