Sl.No. :

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# Fourth Semester M.Sc. Degree Examination, June - 2023

## (Non CBCS) (SLM Scheme)

## MATHEMATICS

## Number Theory

**Time : 3 Hours** **Max. Marks : 80**

**Instructions : 1) Answer any five full questions.**
**2) All questions carry equal marks.**

***Q1)*** a) If $p$ is a prime and $p/ab$ then prove that $p/a$ or $p/b$.

b) Find the *gcd* of 256 and 1166.

c) If $a$ and $b$ are any two integers not both of them are zero then show that there exists integers $x$ and $y$ such that $gcd\ (a, b) = ax + by$.

**[4 + 4 + 8]**

***Q2)*** a) State and prove fundamental theorem of Arithmetic.

b) Explain the prime number theorem.

c) State and prove chiness Remainder theorem.

**[6 + 4 + 6]**

***Q3)*** a) Prove that the quadratic congruence $x^2 + 1 \equiv 0 \pmod{p}$ where $p$ is an odd prime has a solution if and only if $p \equiv 1 \pmod{4}$.

b) If $n \geq 1$ and $gcd\ (a, n) = 1$ then prove that $a^{\phi(n)} \equiv 1 \pmod{n}$.

**[7 + 9]**

***Q4)*** a) If $n = p_1^{k_1}, p_2^{k_2}, ....., p_r^{k_r}$ is the prime factorization of $n > 1$ then prove that :

i) $\tau(n) = (k_1 + 1)\ (k_2 + 2)\ .....\ (k_r + 1)$

ii) $\sigma(n) = \dfrac{p_1^{k_1+1} - 1}{p_1 - 1}\ \dfrac{p_2^{k_2+1} - 1}{p_2 - 1} ..... \dfrac{p_r^{k_r+1} - 1}{p_r - 1}$.

b) State and prove the Mobius inversion formula.

**[8 + 8]**

***Q5)*** a) For any positive integer $n$ then prove that $\phi(n) = n\sum_{d/n} \dfrac{\mu(d)}{d}$.

b) Using the ceaser, cipher, encipher the following "ALL IS WELL THAT ENDS WELL".

c) Define dirichlet product of two arithmetical functions. Prove that dirichlet product is commutative and associative.

**[6 + 5 + 5]**

***Q6)*** a) Show that there exists a primitive root for $p^k$ when $p$ is an odd prime and $k \geq 1$.

b) Solve the congruence of $x^{15} \equiv 7 \pmod{19}$.

c) Construct the index table for 17 with primitive root 5.

**[6 + 5 + 5]**

***Q7)*** a) State and prove the Gauss Lemma for quadratic residue. Evaluate n of Gauss Lemma for (11/23).

b) Show that an odd prime $p$ is expressible as a sum of two squares if and only if $p \equiv 1 \pmod{4}$.

**[10 + 6]**

***Q8)*** a) Define the finite continued fraction show that every rational number can be written as a finite simple continued fraction.

b) Show that $F_n^{\,2} = F_{n+1}\ F_{n-1} + (-1)^{n-1}$.

c) Express $\sqrt{15}$ as infinite simple continued fraction.

**[6 + 5 +5]**

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