Math 4.1

Sl.No. :

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IV Semester M.Sc. Degree Examination, May - 2026

(SLM Scheme) (Non-CBCS)

MATHEMATICS

Number Theory

Time : 3 Hours **Max. Marks : 80**

Instructions : 1) Answer any five full questions.

2) All questions carries equal marks.

Q1) a) If a and b are any two integers, not both of then are zero, then show that these exists integers x and y such that $gcd(a, b) = ax + by$.

b) State and prove the fundamental theorem of arithmetic.

c) Explain the prime number theorem.

[5 + 7 + 4]

Q2) a) State and prove Chinese remainder theorem.

b) State and prove Wilson's theorem. What about the converse? Justify.

[8 + 8]

Q3) a) If $n = p_1^{k_1} p_2^{k_2}, ..., p_r^{k_r}$ is the prime factorization of $n > 1$ then prove that

i) $\tau(n) = (k_1 + 1)\ (k_2 + 1)\ ..... .(k_r + 1)$

ii) $\sigma(n) = \frac{p_1^{k_1+1} - 1}{p_1 - 1} . \frac{p_2^{k_1+1} - 1}{p_2 - 1} ..... \frac{p_r^{k_r+1} - 1}{p_r - 1}$

b) State and prove the Mobius inversion formula.

[8 + 8]

Q4) a) Show that for each positive integer $n \geq 1, n = \sum_{d|n} \phi(d)$, the sum being extended over all positive divisors of n.

b) Explain the Hill Cipher with an example.

c) Show that Dirchlet multiplication is commutative and associative.

[6 + 5 + 5]

Q5) a) Given a prime p, let $f(x) = a_0 + a_1 x + ...+ a_n x^n$; $a_i \in z$ be a polynomial of degree 'n' such that $a_n \neq 0 (\text{mod } p)$. The prove that the polynomial congruence $f(x) \equiv 0 \pmod{p}$ has atmost 'n' solutions.

b) If $m, n > 2$ and $gcd(m, n) = 1$, then prove that there exist no. primitive root (mod mn).

c) Construct the index table for 17 with primitive root 5.

[5 + 6 + 5]

Q6) a) Let p be an odd prime and $gcd(a, p) = 1$. Then prove that 'a' is a quadratic residue of p if and only if $a^{(p-1)/2} \equiv L (\text{mod } p)$.

b) State and prove Gauss Lemma.

c) Evaluate Legendre Symbol $\left(\frac{504}{23}\right)$.

[6 + 8 + 2]

Q7) a) Prove that an odd prime p is expressible as a sum of two squares if and only if $p \equiv 1 \pmod 4$.

b) Explain 1613 as the sum of two squares.

c) Define Fibonacci sequence. Prove that $F_{m+n} = F_{m-1} F_n + F_m F_{n+1}$

[6 + 4 + 6]

Q8) a) Show that every finite simple continued fraction is equal to a rational number.

b) Solve the linear Diophantine equation. $(-63)x + 23y = 7$, where $gcd(-63, 7) = 1$.

c) Express $\sqrt{15}$ as infinite simple continued fraction.

[6 + 5 + 5]

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