Math-4.1

Sl.No. :

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Fourth Semester M.Sc. Degree Examination, May - 2025
(SLM Scheme) (Non - CBCS)
MATHEMATICS
Number Theory

Time : 3 Hours **Max. Marks : 80**

Instructions : ***1) Answer any five full questions.***
2) All questions carry equal marks.

Q1) a) Show that square of any integer is either of the form 3k or 3k+1.

b) Explain the prime number theorem.

c) State and prove the fundamental thorem of arithmetic.

[3 + 5 + 8]

Q2) a) State and prove Fermat's Little Theorem.

b) If p is a prime, then prove that $(p-1)! \equiv -1 \pmod{p}$.

c) Find the solutions of the system of congruences.

$3x + 4y \equiv 5 \pmod{13}$

$2x + 5y \equiv 7 \pmod{13}$

[6 + 4 + 6]

Q3) a) Prove that the quadratic congruence $x^2 + 1 \equiv 0 \pmod{p}$ where p is an odd prime, has a solution if and only if $p \equiv 1 \pmod{4}$.

b) Define Möbius function 'μ'. Prove that, the function 'μ' is multiplicative function.

c) For a positive integer n, prove that $\sum_{d/n} \frac{\mu^2(d)}{\phi(d)} = \frac{n}{\phi(n)}$.

[6 + 5 + 5]

Q4) a) Using the ceaser cipher, encipher the following "ALL IS WELL THAT ENDS WELL".

b) Let n = pq, where p and q are primes with p > q. Show that $p + q = n - \phi(n) + 1$.

c) For $x \geq 1$ prove that $\sum_{n \leq x} \frac{1}{n} = \log x + c + o\left(\frac{1}{x}\right)$.

[6 + 5 + 5]

Q5) a) Show that there exists a primitive root for p^k when p is an odd prime and $k \geq 1$.

b) Solve the congruence : $2x^4 \equiv 5 \pmod{13}$.

c) Construct the index (mod 18).

[6 + 5 + 5]

Q6) a) State and prove Gauss Lemma.

b) Let p be an odd prime and gcd (a, p) = 1. Then show that 'a' is a quadratic residue of p if and only if $a^{(p-1)/2} \equiv 1 \pmod{p}$

c) Evaluate Legendre symbol $\left(\frac{-23}{59}\right)$.

[8 + 6 + 2]

Q7) a) State and prove quadratic reciprocity law.

b) Express 541 as sum of two squares.

c) Show that, $F_1 + 2F_2 + 3F_3 + \ldots + nF_n = (n + 1) F_{n+2} - F_{n+4} + 2$.

[6 + 5 + 5]

Q8) a) Prove that every finite simple continued fraction is equal to a rational number and every rational number can be written as a finite simple continued fraction.

b) If the number '*x*' has a periodic simple continued fraction expansion, then show that *x* is a quadratic irrational.

c) Express $\sqrt{7}$ as infinite simple continued fraction.

[6 + 6 + 4]

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