Sl.No. :

Total No. of Pages : 3

Fourth Semester M.Sc. Degree Examination, June - 2023

(Non CBCS) (SLM Scheme)

MATHEMATICS

Graph Theory and Algorithms

Time : 3 Hours **Max. Marks : 80**

Instructions : 1) Answer any five questions.

2) All questions carry equal marks.

Q1) a) Prove that the sum of the degree of the vertices of a graph G is twice the number of edges. Further, prove that every cubic graph has even number of vertices.

b) Define a path show that every *uv* walk in a graph contains a *u*–*v* path.

c) Prove that with usual notation, $\delta(G) \leq \frac{2m}{n} \leq \Delta(G)$.

[6 + 4 + 6]

Q2) a) If B(G) is an incidence matrix of a connected graph G with *n* vertices, then prove that rank of B(G) is $(n - 1)$.

b) Prove that a simple graph G with *n* vertices and *k* components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges.

c) Let G be a labeled graph with adjacency matrix A. Then show that the $(i, j)^{th}$ entry of A^n is the number of walks of length *n* from vertex v_i to v_j.

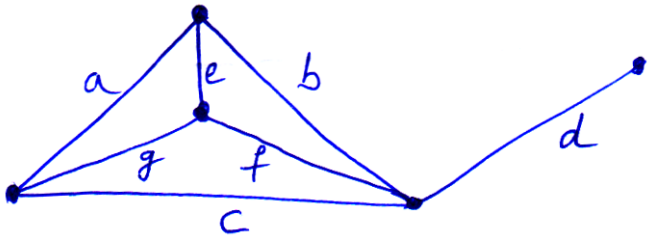
[6 + 5 + 5]

Q3) a) Define isomorphism of graphs. Prove that the following graphs are isomorphic.



b) Define complement of a graph. Prove that any self-complementary graphs has 4n or 4n+1 vertices.

c) Find the cut-set matrix of the graph given below.



[5 + 6 + 5]

Q4) a) Prove that a connected graph G has an Euler circuit if and only if all vertices of G are of even degree.

b) Prove that if G is a simple (n, m) graph with at least three vertices and $\delta(G) \geq \frac{n}{2}$ then G is Hamiltonian.

[8 + 8]

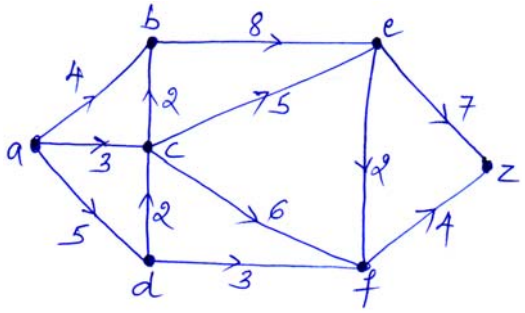
Q5) a) Prove that for a sequence $d_1, d_2, \ldots\ldots\ldots, d_n$ $(n \geq 2)$ of positive integers is the degree sequence of a tree of order n if and only if $\sum_{i=1}^{n} di = 2n - 2$.

b) Show that a graph with atleast $2n$-vertices is n-connected if and only if for any two disjoint sets V_1 and V_2 of n-vertices each, there exist 'n' disjoint paths joining these two sets of vertices.

[8 + 8]

Q6) a) Explain the Breadth first search algorithm with an example.

b) Apply Dijkstra's algorithm to the weighted digraph shown in the following figure in order to find the shortest distance from the vertex *a* to *z*.



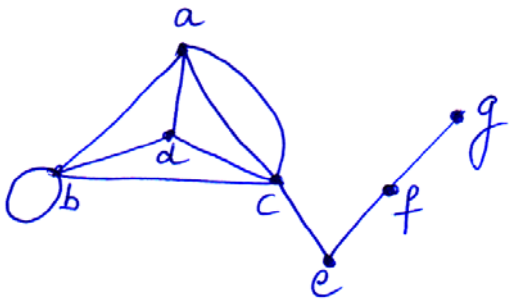
[8 + 8]

Q7) a) Define planar graph verify whether the complete bipartite graph $K_{3,3}$ is planar.

b) Prove that a graph can be embedded in the surface of a sphere if and only if it can be embedded in a plane.

[8 + 8]

Q8) a) Draw the geometric dual of the graph.



b) State and prove Havel-Hakimi theorem.

[6 + 10]

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