Math 4.2

Sl.No. :

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IV Semester M.Sc. Degree Examination, May - 2026
(SLM Scheme) (NON CBCS)
MATHEMATICS
Math 4.2 : Graph Theory and Algorithms

Time : 3 Hours **Max. Marks : 80**

Instructions : 1) Answer any five full questions.

2) All questions carry equal marks.

Q1) a) Prove that the sum of the degree of the vertices of a graph G is twice the numbers of edges. Furhter prove that every cubic graph has even number of vertices.

b) Prove with usual notation $\delta(\mathrm{G}) \leq \frac{2m}{n} \leq \Delta(G)$.

c) Define spanning and induced subgraphs. Distinguish between edge-disjoint and vertex disjoint subgraphs.

[6+6+4]

Q2) a) Define the following with an example :

i) Composition product

ii) Normal product

iii) Tensor product

b) Let G be a labeled graph with adjacency matric A. Then show that the $(i,j)^{\text{th}}$ entry A^n is the number of walks of length n from v_i to v_j.

c) If X is a cut-set matrix of a connected graph G with 'e' edges and n-vertices, then show that the rank of X = $n - 1$.

[6+5+5]

Q3) a) If G has a Hamiltonian cycle, then show that for each non-empty set $S \subseteq V(G)$, the graph G-S has atmost |S| components.

b) Explain the following :-

i) Seating problem

ii) Travelling salesman problem

c) Show that a connected graph is a tree if and only if it is minimally connected.

[6+4+6]

Q4) a) Show that any graph G is connected if and only if it has a spanning tree.

b) Show that every cut-set in connected graph G must contain atleast one branch of every spanning tree of G.

c) Define the following.

i) Block graph

ii) Cut-Vertex graph.

[6+6+4]

Q5) a) Show that for any graph G, $K(G) \leq \lambda(G) \leq \delta(G)$.

b) State and prove the Menger's theorem.

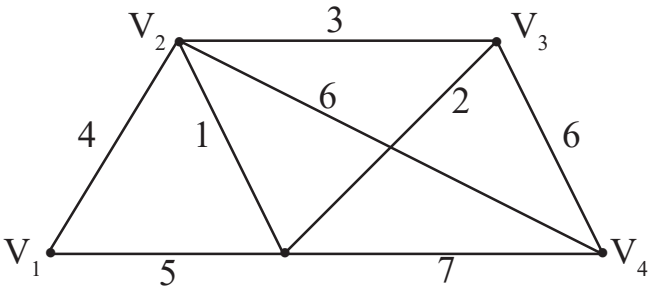
[6+10]

Q6) a) Explain the following with an example.

i) Pre-order and post-order traversals.

ii) Depth first search algorithm.

b) Using Prim's algorithm, find a minimal spanning tree for the weighted graph.



[10+6]

Q7) a) Define planar graph. Verify whether the complete bipartite graph $K_{3,3}$ is planar.

b) Prove that a graph can be embedded in the surface of a sphere if and only if it can be embedded in a plane.

[8+8]

Q8) a) State and prove the Havel-Hakimi theorem.

b) Show that a graph has a dual if and only if it is planar.

[10+6]