Math - 4.2

Sl.No. :

Total No. of Pages : 3

IV Semester M.Sc. Degree Examination, May - 2025
(SLM Scheme)
MATHEMATICS (NON-CBCS)
Graph Theory and Algorithms

Time : 3 Hours **Max. Marks : 80**

Instructions : 1) Answer any five questions.

2) All questions carry equal marks.

Q1) a) Prove that every u-v walk in a graph contains a u-v path.

b) Show that the maximum number of edges in a bipartite graph of order 2n is n^2.

c) Prove that a simple graph G with P vertices and K components can have atmost $\frac{(p-k)(p-k+1)}{2}$ edges.

[6+5+5]

Q2) a) Prove that a graph is bipartite if and only if all its cycles are even.

b) Prove that in a connected graph G, distance is metric on V(G).

c) Define adjacency matrix A(G) of a labeled graph G. Let G be a labeled graph with adjacency matrix A. Then prove that (i,j) th entry of A^n is the number of walks of length n from v_i to v_j.

[6+5+5]

Q3) a) State and prove Ore's theorem on Hamiltonian graph.

b) Prove that a sequence $d_1, d_2, \ldots d_n$ $(n \geq 2)$ of positive integers is the desier sequence of a tree of order n if and only if $\sum_{i=1}^{n} d_i = 2n-2$.

[8+8]

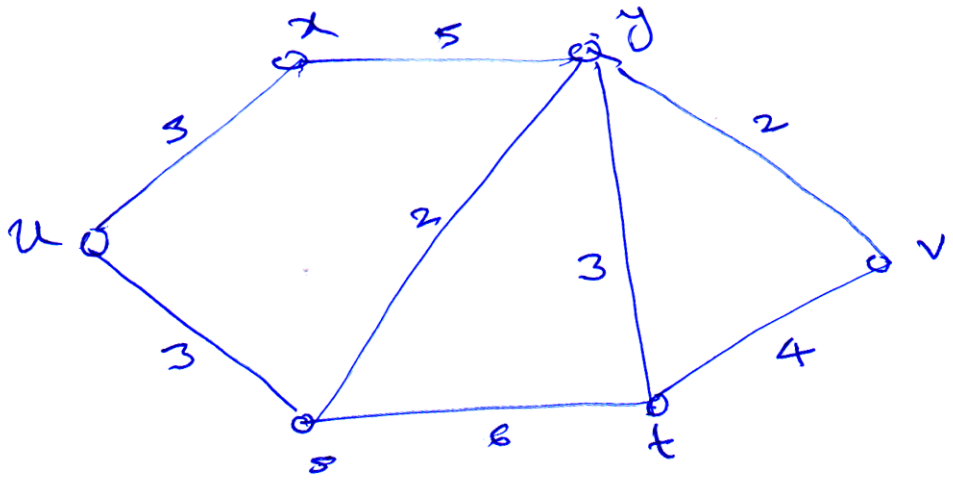
Q4) State and prove Menger's theorem. **[16]**

Q5) a) Explain the following with an example

i) Pre-order and post -order traversals.

ii) Depth first search algorithm.

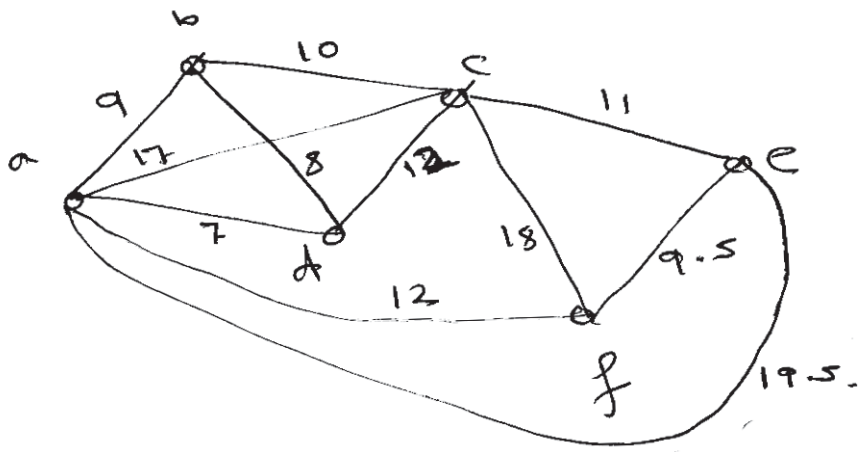
b) Using Dijkstra's algorithm find the shortest path between u and v in the weighted graph given below



[8+8]

Q6) a) Explain the minimal spanning tree with an example.

b) Using prim's algorithm, find minimal spanning tree of the weighted graph shown below.



[6+10]

Q7) a) Prove that the complete graph on 5 vertices and complete bipartite graph $K_{3,3}$ are non planar graphs.

b) Prove that a connected planer graph with n vertices and e edges has e-n+2 regions.

c) Prove that a graph is planer if and only each of its blocks in planer.

[6+5+5]

Q8) a) Show that the partition P = (6,6,5,4,3,3,1) is not graphic.

b) Prove that a sequence $P = (d_1, d_2, ..., d_p)$ of non negative integers with $d_1 \geq d_2 \geq ..., \geq d_p$, $p \geq 2$, $d_1 \geq 1$, is graphical if and only if the sequence $P_1 = (d_2 - 1, d_3 - 1, ..., d_{di+1} - 1, d_{i+2} ....d_p)$ is graphical. **[6+10]**

