Math-4.4

Sl.No. :

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Fourth Semester M.Sc. Degree Examination, June - 2023
(Non CBCS) (SLM Scheme)
MATHEMATICS
Mathematical Statistics

Time : 3 Hours **Max. Marks : 80**

Instructions : 1) Answer any five full questions.
2) All questions carry equal marks.
3) Use of scientific calculator is permitted.

Q1) a) Explain Axiomatic definition of probability measure. For any sequence (A_n) of events prove that $P(\bigcup_n A_n) \leq \sum_n P(A_n)$.

b) If A, B, C are the events with $P(A) = P(B) = P(C) = \frac{1}{6}$; $P(A \cap B) = P(B \cap C) = \frac{1}{6}$, $P(A \cap C) = \frac{1}{6}$ and $P(A \cap B \cap C) = \frac{1}{20}$. Find $P(A \cup B \cup C)$.

c) Let $A_1, A_2, A_3, \ldots\ldots$ be disjoint events in the space (Ω, H, P) Then, prove that $P(\bigcup_j A_j / B) = \sum_j P(A_j / B)$.

[8 + 4 + 4]

Q2) a) State Baye's theorem. Illustrate the use of Baye's theorem to solve the following problem. In a factory, machines A, B, C produce respectively 25%, 35% and 40% of the total production. Of which 5%, 4% and 2% are defective. An item is drawn at random was found to be defective. Find the probability that

i) it was manufactured by A.

ii) it was manufactured by B or C.

b) Define a random variable and elementary random variable. If the selection for a job is based on the pooled percentage of marks in the entrance test and viva, there is a weightage of 0 if the score is in [0, 35); 1 if it is in [35, 50); 3 if it is in [50, 60) and 6 if it is in [60, 100]. Write the sample space and give the associated random variable.

c) Prove that, every distribution function F has at most a countable set of discontinuity points. Moreover, show that $F = F_c + F_d$, where F_c is a continuous function and F_d is a pure step function.

[6 + 5 + 5]

Q3) a) State and prove Jordan decomposition theorem.

b) Let X be a Poisson random variable with $pmf\ p_k = e^{-\lambda}\frac{\lambda^k}{k!},\ k = 0,1,2,.....$ Then, show that EX = λ.

c) Let X be a random variable with $EX^2 < \infty$ and let *t* be any positive constant. Then prove that $P(|X| > t) \leq \frac{EX^2}{t^2}$.

[8 + 4 + 4]

Q4) a) The probability that a book manufactured by a company will be defective is $\frac{1}{20}$. If 24 such books are manufactured, find the probability that :

i) exactly two will be defective

ii) at least two will be defective

iii) none will be defective

b) Let X be a Normal random variable with probability density function $f(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-(x-\mu)^2/2\sigma^2}$, show that the mean of the distribution is μ and the variance is σ^2,

[6 + 10]

Q5) a) Establish that if $X_1 \sim N(0.1)$ then $X_1^2 \sim X_1^2$ distribution.

b) If $\{X_n, n \geq 1\}$ is a sequence of independent and identically distributed random variables, then prove that, the random variable $\frac{S_n}{n} = \frac{X_1 + X_2 + ... + X_n}{n}$ converges in probability to $E(X_1)$ provided $E(X_1^2) < \infty$.

[8 + 8]

Q6) a) Define the convergence in distribution and the convergence in probability. Prove that $X_n \xrightarrow{P} X \Rightarrow X_n \xrightarrow{d} X$.

b) State and prove Neyman factorization theorem.

[8 + 8]

Q7) a) Describe one sample T-test.

b) Describe the flexibility of Chi-square test.

[8 + 8]

Q8) a) Find confidence interval of $(\mu_2 - \mu_1)$ for large sample.

b) Let (X, Y) be two dimensional random variable. If the regression of Y on X is linear then prove that $E(Y / X) = \bar{Y} + r\frac{\sigma_x}{\sigma_y}(X - \bar{X})$.

[8 + 8]

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