MATH 4.4

Sl.No. :

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Fourth Semester M.Sc. Degree Examination, May - 2025
(SLM Scheme) (Non CBCS)
MATHEMATICS
Mathematical Statistics

Time : 3 Hours **Max. Marks : 80**

Instructions: **1) Answer any five full questions.**

2) All questions carry equal marks.

1. a) Define a field prove that intersection of a finite number of fields, defined over a space Ω, is also a field.

b) For any sequence (A_n) of events prove that,

i) $P(\textit{lim inf}\ A_n) \leq \textit{lim inf}\ P(A_n)$

ii) $P(\textit{lim sup}\ A_n) \geq \textit{lim sup}\ P(A_n)$

c) Let (A_n) be a sequence of events with $P(A_n) = \frac{1}{3^n}$, $n \geq 1$, obtain upper bounds for $(p\bigcup_{n=1}^{10} A_n)$ and $P\left(\bigcup_{n=1}^{\infty} A_n\right)$.

[6 + 6 + 4]

2. a) State and prove Baye's theorem.

b) Prove that the sum, difference and product of elementary random variables will again be elementary random variables.

c) If X is a random variable then show that X^2 is also a random variable.

[6 + 6 + 4]

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3. a) State and prove Jordan decomposition theorem.

b) Let X be a Poisson random variable with pmf $Pk = e^{-\lambda}\frac{\lambda^k}{K!}$, $k = 0,1,2,.....$ then, show that $Ex = \lambda$.

c) Let X be a random variable with $Ex^2 < \infty$ and let t be any positive constant. Then prove that $P(1 \times 1 > t) < \frac{Ex^2}{t^2}$.

[8 + 4 + 4]

4. a) Write the probability generating function of Binomial distribution and show that mean $E(X) = np$ and variance $\sigma^2 = npq$.

b) In a normal distribution 31% of the items are under 45 and 8% of the items are over 64. Find the mean and the standard deviation of the distribution.

[8 + 8]

5. a) Let X, Y, Z be independent random variables with unit exponential distribution. Find the distributions of the random variables U = X + Y + Z, $V = \frac{X+Y}{X+Y+Z}$ and $W = \frac{X}{X+Y}$.

b) Establish that if $X, \sim N(0,1)$ then $X_1^2 \sim X_1^2$ distribution. **[8 + 8]**

6. a) Define the convergence in distribution and the convergence in probability. Prove that $X_n \xrightarrow{p} X \Rightarrow X_n \xrightarrow{d} X$.

b) State and prove Neyman factorigation theorem.

[8 + 8]

7. a) Define most powerful test and UMP test. State Neyman-Pearson lemma.

b) Describe Chi-Square test for two independent experiments.

[6 + 10]

8. a) Find confidence internal of $(\mu_2 - \mu_1)$ for large samples.

b) Let (X, Y) be two dimensional random variable. If the regression of Y on X is linear then prove that $E(X \mid Y) = \overline{X} + r\frac{\sigma_y}{\sigma_x}(Y - \overline{Y})$

[8 + 8]

