MMDSC-1.1

Sl. No.

I Semester M.Sc. Degree Examination, December - 2023
MATHEMATICS
Algebra - I (Course - I)
(CBCS Scheme)

Time : 3 Hours **Maximum Marks : 80**

***Instructions to Candidates*:**

Answer any ***Five full*** questions. ***All*** questions carry equal marks.

1. a) State and prove the Lagrange theorem for finite groups. **(4+4+8)**

b) Show that a subgroup N of G is a normal subgroup of G if and only if the product of two right cosets of N in G is again a right coset of N in G.

c) Define types of homomorphism.

Let $\phi : G \to G$ ' be an onto group homomorphism with $\text{Ker}\,\phi = K$. Then show that G' $\cong G / K$.

2. a) If N, M are normal subgroups of a group G, prove that $NM/M \approx N / N \cap M$. **(5+6+5)**

b) Define a group of an inner automorphism $Inn(G)$ of a group. Prove that $Inn(G) \approx G / Z(G)$, where $Z(G)$ is the centre of G.

c) Prove that any non-abelian group of order 6 is isomorphic to symmetric group S_3.

3. a) Derive the Class equation of finite group. **(8+8)**

Further, verify the Class equation of Symmetric group S_3.

b) Show that for any group of order p^n, where p is prime, has nontrivial center. Further, show that $o(G) = p^2$ is abelian.

4. a) Define p-sylow subgroups of a finite group. Prove that any two p-sylow subgroups of same order are conjugate to each other. **(7+5+4)**

b) Let $o(G) = pq$, where p and q are distinct primes with $p<q$ and $q \neq 1 \pmod{p}$, then prove that G is cyclic and abelian.

c) Show that a group of order $13^2.17^2$ is abelian.

[P.T.O.

5. a) Let R be a commutative ring with unity whose ideals are $\{0\}$ and R itself. Then show that R is a field. **(4+4+8)**

b) Let U be a left ideal of a ring R and $\lambda(U) = \{x \in R / xu = 0 \ for\ all\ u \in U\}$. Then show that $\lambda(U)$ is an ideal of R.

c) Let R and R' be rings and ϕ is homomorphism of R onto R' with kernel U. Then prove that R' $\approx R/U$. Further, show that there is one to one correspondence between the set of ideals W' of R'and the set of ideals $W$of R containing U.

6. a) Show that a commutative ring R with identity and an ideal P in R is a prime ideal if and only if R/P is an integral domain. **(5+5+6)**

b) Show that for any commutative ring with unity, a maximal ideal is always a prime ideal.

c) If K is any field containing an integral domain D, then K contains a subfield isomorphic to the quotient field $F(D)$.

7. a) Define an Euclidean ring. Prove that **(8+4+4)**

i. Every field is an Euclidean ring

ii. The ring $Z[i]$ of Gaussian integers is an Euclidean ring.

b) State and prove the Unique factorization theorem.

c) If p is a prime number of the form $4n+1$, then prove that $p=a^2+b^2$ for some integers a and b.

8. a) Define a primitive polynomial of a ring. **(6+10)**

Prove that the product of primitive polynomials is a primitive polynomial.

b) State and prove the Eisenstein criterion for the irreducibility of a polynomial. Further, verify that $f(x) = x^3 + x^2 - 2x - 1 \in Q[x]$ is irreducible polynomial, by using Eisenstein criteria.
