MMDSC-1.1

Sl.No. :

Total No. of Pages : 2

I Semester M.Sc. Degree Examination, June/July - 2024

(CBCS, SLM Scheme)

MATHEMATICS

Algebra - I

Time : 3 Hours **Max. Marks : 80**

Instructions: ***1) Answer any five questions.***
2) All questions carry equal marks.

Q1) a) Define a subgroup of a group. Let H and K be subgroups of a group G. Then prove that HK is a subgroup of G, if and only if HK = KH.

b) State and prove Lagrange theorem for finite groups.

c) State and prove fundamental theorem of homomorphism for groups.

[5+5+6]

Q2) a) State and prove first isomorphism theorem.

b) Define inner automorphism. If G is a group then prove that the set I*nn* (G) of all inner automorphism of G is also a group.

c) State and prove the Cayley's theorem for permutation.

[6+5+5]

Q3) a) Derive the class equation of finite group.

b) Every P group has a non-trivial centre.

c) Let G be a finite group and *x, y* be conjugate elements of G. Then show that the number of distinct elements $g \in G$ such that $g^{-1}xg = y$ is O(N(x)).

[6+4+6]

Q4) a) State and prove third sylow theorem.

b) Using sylow's theorems, show that (P-1)! ≡ -1(modP) for any prime P.

c) Show that any group of order $5^2 . 7^2$ is abelian.

[8+4+4]

Q5) a) Let R be a commutative ring with unity. Then prove that R is a field it and only if the only ideals if R are {0} and R it self.

b) State and prove fundamental theorem of homomorphism for rings.

c) Let U is an ideal of R and a set [R:U] = $\{x \in R : rx \in U \ \forall r \in R\}$ then prove that [R:U] is an ideal of R containing U.

[5+7+4]

Q6) a) Show that every integral domain can be embedded in a field.

b) Prove that in a commutative ring with unity, a maximal ideal is always a prime ideal.

[10+6]

Q7) a) Show that $Z(\sqrt{2}) = \{a + \sqrt{2}b : a, b \in z\}$ is an euclidean domain.

b) State and prove Fermat's theorem on sums of squares.

[10+6]

Q8) a) If F is a field, then prove that F[x] is an euclidean ring.

b) State and prove Eisensten's criterion for irreducibility of a polynomial.

c) Prove that $f(x) = 2x^2 - 4x + 6$ is irreducible over Q.

[6+8+2]