MMDSC-1.1

Sl.No. :

Total No. of Pages : 2

I Semester M.Sc. Degree Examination, May/June - 2023

(CBCS, SLM Scheme)

MATHEMATICS

Algebra - I

Time : 3 Hours **Max. Marks : 80**

Instruction: ***1) Answer any five full questions.***
2) All questions carry equal marks.

Q1) a) Define a subgroup of a group. Let H and K be subgroup of a group G then prove that HK is a subgroup of G, if and only if HK = KH.

b) Let G be a finite group and H a subgroup of G. Then show that the order of H divides the order of a group G.

c) State and prove fundamental theorem of homomorphism for groups.

[6+5+5]

Q2) a) State and prove first isomorphism theorem.

b) Define inner automorphism. If G is a group then prove that the set Inn (G) of all inner automorphism of G is also a group.

[8+8]

Q3) a) Derive the class equation.

b) Let $H \neq G$ be a subgroup of a finite group G. Then show that G cannot be expressed as union of conjugates of H.

c) Suppose X is a conjugate class of non-trivial element of a group G. Let $T \in Aut(G)$. Then show that $T(X) = \{T(x) | x \in x\}$ is a conjugate class of elements of G.

[6+4+6]

Q4) a) State and prove third Sylow theorem.

b) Let O(G) = pq where p and q are prime with $p < q$ and $q \equiv 1 (\text{mod } p)$ then prove that G is non-abelian.

c) Show that any group of order $5^2 \cdot 7^2$ is abelian.

[6+6+4]

Q5) a) State and prove fundmental theorem of homomorphism for rings.

b) Let R be a ring with unity then prove that R is a division ring if and only if the only ideals are R and $\{0\}$.

c) Let U is an ideal of R and a set $[R : U] = \{x \in R : rx \in U \ \forall \ r \in R\}$ then prove that [R:U] is an ideal of R containing U.

[7+5+4]

Q6) a) If R is a commutative ring with unity and M is an ideal of R then prove that M is a maximal ideal of R if $\frac{R}{M}$ is a field.

b) Show that every integral domain can be embedded in a field.

[6+10]

Q7) a) Define an Euclidean Domain. Show that every Euclidean ring is a principal ideal ring.

b) State and prove unique factorisation theorem.

[8+8]

Q8) a) State and prove Eisensten's criterion for irreducibility of a polynomial.

b) If F is a field then F[x] is an Euclidean ring.

c) Prove that $f(x) = 2x^2 - 4x + 6$ is irreducible over Q.

[8+6+2]

