MMDSC-1.1

Sl.No. :

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I Semester M.Sc. Degree Examination, May/June-2026
(SLM Scheme, CBCS)
MATHEMATICS
Course - I : Algebra - I

Time : 3 Hours **Max. Marks : 80**

Instruction : ***Answer any five full questions.***

Q1) a) Let H and K are two subgroups of a group G. Then show that HK is a subgroup of G if and only if HK = KH.

b) Explain the converse of Lagrange's theorem with suitable example.

c) State and prove fundamental theorem of homomorphism.

[6+4+6]

Q2) a) State and prove second isomorphism theorem.

b) Prove that $\mathrm{Inn(G)} \cong \frac{\mathrm{G}}{z(\mathrm{G})}$, where Inn(G) is the group of inner automorphisms of G and z(G) is the centre of G.

c) State and prove Cayley's theorem.

[6+4+6]

Q3) a) Prove that, for a finite group G, $\mathrm{O(G)} = \mathrm{O}\left(z(\mathrm{G})\right) + \sum_{\mathrm{N}(a)\neq \mathrm{G}} \frac{\mathrm{O(G)}}{\mathrm{O}\left(\mathrm{N}(a)\right)}$.

b) State and prove Cauchy's theorem.

c) Show that an abelian group of order 15 is Cyclic.

[6+6+4]

Q4) a) State and prove Sylow's second theorem.

b) Let G be group of order 231. Show that H-Sylow subgroup of G is contained in the centre of G.

c) Show that group of order $5^2.7^2$ is abelian.

[7+4+5]

Q5) a) Show that the ring of real queternions is a division ring.

b) State and prove fundamental theorem of homomorphism of rings.

c) Let A, B be two ideals of a ring R. Show that $\frac{A+B}{A} \cong \frac{B}{A \cap B}$.

[4+6+6]

Q6) a) If R is a commutative ring with unity and M is an ideal of R, then prove that M is a maximal ideal of R if and only if R/M is a field.

b) Let R be an integral domain with ideal P, then prove that p is a prime ideal if and only if R/P is an integral domain.

[8+8]

Q7) a) Prove that every Euclidean ring is a principle ideal ring.

b) Show that Z[i] is an Euclidian domain.

c) Show that $z\left[\sqrt{-5}\right]$ is an integral domain, which is not an unique factorization domain.

[6+5+5]

Q8) a) State and prove Eisenstein criteria for irreducibility of a polynomial.

b) State and prove Gauss Lemma.

[8+8]

