MMDSC-1.1

Sl.No. :

Total No. of Pages : 2

I Semester M.Sc. Degree Examination, June-2025
(CBCS, SLM Scheme)
MATHEMATICS
Course - I: Algebra - I

Time : 3 Hours **Max. Marks : 80**

Instruction : Answer any five full questions. All questions carry equal marks.

Q1) a) Let H be a subgroup of a finite group G. Then show that the order of H divides the order of G.

b) Define a Normal subgroup of a group.

The subgroup N is a normal subgroup of a group G if and only if every left coset of N in G is a right coset of N in G.

c) State and prove the fundamental theorem of homomorphism for groups.

[5+6+5]

Q2) a) Let H and K be normal subgroups of a group G such that $K \subseteq H$. Then show that H / K is normal in G / K and (G / K) / (H / K) $\cong$ G / H.

b) Show that a homomorphism T: G$\rightarrow$G defined by T(x) = x^{-1} is an automorphism if and only if G is an abelian.

c) State and Prove Cayley's Theorem.

[6+5+5]

Q3) a) If H is a subgroup of G, let N (H) = $\{g \in G / gHg^{-1} = H\}$. Show that,

i) N(H) is a subgroup G.

ii) H is a normal in N(H).

b) Derive the class equation of finite groups. Further, verify the class equation of symmetric group S_3.

c) Show that for any group of order p^n, where p is prime, has nontrivial center.

[4+8+4]

Q4) a) State and prove First Sylow's theorem.

b) Let o(G) = pq, where p and q are distinct primes with $p < q$ and $q \neq 1 (\text{mod } p)$, then prove that G is cyclic and hence abelian.

[10+6]

Q5) a) Let R be a commutative ring with unity whose ideals are {0} and R itself. Then show that the commutative ring R is a field.

b) Show that the homomorphism ϕ: R$\rightarrow$R' is an isomorphism if and only if Ker(ϕ)={0}.

c) Let R and R' be rings and ϕ is homomorphism of R onto R' with kernel U. Prove that R'$\approx$R/U.

[5+5+6]

Q6) a) Define an ideal of a ring R with suitable example.

Let U be a left ideal of a ring R and λ(U) = {$x \in$ R / xu=0 for all $u \in$ U}. Then show that λ(U) is an ideal of R.

b) Show that a ring Z of integers is a principal ideal ring

c) If R is a commutative ring with unity and M is an ideal of R, then show that M is a maximal ideal of R if and only if R/M is a field.

[6+4+6]

Q7) a) Define an Euclidean ring of a ring R with suitable example. Show that every Euclidean ring is a principle ideal ring.

b) State and prove the Unique factorization theorem.

c) If p is a prime number of the form $4n+1$, then show that $x^2 \equiv -1 \text{ (mod } p)$.

[6+6+4]

Q8) a) If the ring R is an integral domain, then show that R[x] is also an integral domain.

b) Show that the product of two primitive polynomials is a primitive.

c) State and prove the Gauss lemma.

[5+6+5]

ⓘⓘⓘⓘ