MMDSC-1.2

Sl. No.

I Semester M.Sc. Degree Examination, December - 2023
MATHEMATICS
Real Analysis - I
(CBCS Scheme)

Time : 3 Hours **Maximum Marks : 80**

***Instructions to Candidates*:**

Answer any ***Five full*** questions.

1. a) Define a countable and uncountable set. **(6+4+6)**

Let $\{A_n\}_{n=1}^{\infty}$ be a countable collection of countable sets then prove that $\bigcup_{n=1}^{\infty} A_n$ is countable.

b) State and prove Cauchy Schwarz inequality.

c) Let A be a countable set and let $A^n = \{(a_1, a_2, ...a_k, ..., a_n) / a_k \in A,$ for $1 \le k \le n$ be the set of all n-tuples for any $n \in \mathbb{N}$. Then prove that A^n is countable.

2. a) Define open set. Let $(x.d)$ be a metric space then prove that every neighbourhood is an open set. **(4+5+7)**

b) If $\{I_n\}$ be sequence of intervals i.e, $I_n=[a_n,b_n]$ in $\mathbb{R}$ such that $I_n \supseteq I_{n+1}$ for each $n \in \mathbb{N}$ then prove that $\bigcap_{n=1}^{\infty} I_n$ is not empty.

c) Show that every k-cell is compact.

3. a) Show that with the example a bounded monotonic sequences is convergent. **(8+8)**

b) Prove that $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$ .Also show that e is irrational.

4. a) State and prove Cauchy's first limit theorem. **(8+8)**

b) Let $\sum_{n=0}^{\infty} a_n$ and $\sum_{n=0}^{\infty} b_n$ be two series then prove that $\sum_{n=0}^{\infty} c_n$ is the product of the two given series $\left(put \quad c_n = \sum_{k=0}^{\infty} a_k b_{n-k}, n = 0,1,2,... \right)$.

[P.T.O.

5. a) Suppose $E \subseteq X$, a metric space p is a limit point of E, f and g are complex functions on E and $\lim_{x\to p} f(x) = A$, $\lim_{x\to p} g(x) = B$. Then prove the following. **(8+8)**

i. $\lim_{x\to p} f(x) \pm g(x) = A \pm B$

ii. $\lim_{x\to p} f(x)\, g(x) = AB$

iii. $\lim_{x\to p} \frac{f(x)}{g(x)} = \frac{A}{B}$

b) Define uniform continuity. Let f be a continuous mapping of a compact metric space X into a metric space Y. Then prove that f is uniform continuous on X.

6. a) Define pathwise connected show that every pathwise connected set S in $\mathbb{R}^n$ is connected. **(8+8)**

b) If f is monotonically increasing on (a,b) then $f(c^+)$ and $f(c^-)$ both exists for each c in (a,b) and we have $\underset{a<t<c}{Sup}\, f(t) = f(c^-) \le f(c) \le f(c^+) = \inf_{c<t<b} f(t)$ further more, if $a<x<y<b$ then show that $f(c^+) \le f(y^-)$.

7. a) Let $f.g : J \to \mathbb{R}$ be differentiable at $c \in J.$ then prove the following: **(8+8)**

i. f+g is differentiable at c with $(f+g)'c = f'(c) + g'(c)$

ii. αf is differentiable at c with $(\alpha f)'c = \alpha f'(c)$

iii. fg is differentiable at c with $(fg)'c = f(c)g'(c) + g(c)f'(c)$

b) Define local maximum and local minimum. Let $J \subseteq \mathbb{R}$ be an interval let $f : J \to \mathbb{R}$ be differentiable on J and $c \in J$. If C is a local extremum of f then *P.T* $f'(c) = 0$.

8. a) State and prove Taylor's theorem. **(8+8)**

b) Evaluate

i. $\lim_{x\to\infty} \frac{x^n}{e^x}$

ii. $\lim_{x\to 0^+} x^{-n} e^{-1/x}$
