MMDSC-1.2

Sl.No. :

Total No. of Pages : 3

I Semester M.Sc. Degree Examination, June/July - 2024

(CBCS, SLM Scheme)

MATHEMATICS

Real Analysis - I

Time : 3 Hours **Max. Marks : 80**

Instruction: ***Answer any five full questions.***

Q1) a) Define ordered set. Show that the set Q does not have the least upper bound property.

b) Prove that for every real $x>0$ and every intiger $n>0$ there is one and only one real Y such that $y^n = x$.

c) Let A be the set of all sequences whose elements are the digits 0 and 1 then prove that set A is uncountable.

[6+6+4]

Q2) a) Define countable set then prove the following

i) Every subset of a countable set A is countable

ii) $\mathbb{N}\times\mathbb{N}$ is countable.

b) Let $\{E_\alpha\}_{\alpha\in I}$ be a collection of sets E_α for each $\alpha\in I$ (I is any finite or infinite set) then prove that $\left(\bigcup_{\alpha\in I} E_\alpha\right)^C = \bigcap_{\alpha\in I}\left(E_\alpha{}^C\right)$.

c) Define connected set. Show that A subset E of the real line $\mathbb{R}$ is connected if and only if it has the following property: If $x\in E, y\in E$ and $x<z<y$ then $z\in E$.

[6+4+6]

Q3) a) Show that the subsequential limits of a sequence $\{x_n\}$ from a closed subset of $\mathbb{R}$.

b) If $\{x_n\}$ and $\{y_n\}$ are sequences of real numbers then prove the following.

i) $\limsup_{n\to\infty}(x_n + y_n) \le \limsup_{n\to\infty}(x_n) + \limsup_{n\to\infty}(y_n)$

ii) $\liminf_{n\to\infty}(x_n + y_n) \ge \liminf_{n\to\infty}(x_n) + \liminf_{n\to\infty}(y_n)$.

[8+8]

Q4) a) Prove that the series $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^P}$ converges if P>1 and diverges if P$\le$1.

b) If $\sum_{n=0}^{\infty} a_n$ converges absolutely such that $\sum_{n=0}^{\infty} a_n = A$ and if $\sum_{n=0}^{\infty} b_n = B$ then prove that $\sum_{n=0}^{\infty} C_n = AB$, where $\sum_{k=0}^{n} a_k b_{n-k}$.

[8+8]

Q5) a) Let 'P' be an limit point of A and assume that $\lim_{x\to p} = a$, $\lim_{x\to p} g(x) = b$ then prove the following.

i) $\lim_{x\to p}\left(f(x) + g(x)\right) = a + b$

ii) $\lim_{x\to p} \lambda f(x) = \lambda a,$ for every real number λ

iii) $\lim_{x\to p} f(x)\cdot g(x) = a.b$.

b) Suppose f is a continuous mapping of a compact metric space Y then prove that $f(x)$ is compact subset of Y.

c) State:

i) Bolzano's theorem

ii) Uniform continuity.

[8+4+4]

Q6) a) Define connectedness S.T. A metric space X is connected if and only if every two valued function on X is connected.

b) Show that every open connected set in $\mathbb{R}^n$ is path wise connected.

c) Define discontinuity and give an example for each type of discontinuties.

[8+4+4]

Q7) a) Let $f : J \to \mathbb{R}$ be differentiable and $f(J) \subset J_1$ an interval and if $g : J_1 \to \mathbb{R}$ is differentiable at $f(c)$, then prove that $g \circ f$ is differentiable at c with $(g \circ f)' c = g'(f(c)).\ f'(c)$.

b) States the followings.

i) Rolle's theorem

ii) Mean value theorem

iii) Inverse function theorem

iv) Cauchy's mean value theorem

[8+8]

Q8) a) State and prove Leibniz formula.

b) Evaluate $\lim_{x\to\infty} \frac{x^n}{e^x}$.

c) Let $f(x) = \begin{cases} e^{-1/x}. \text{ if } n > 0 \\ 0 \quad \text{ if } x \le 0 \end{cases}$ then Evaluate $f'(0)$.

[10+4+2]