MMDSC-1.2

Sl.No. :

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I Semester M.Sc. Degree Examination, May/June - 2026

(SLM Scheme, CBCS)

MATHEMATICS

Course - II : Real Analysis-I

Time : 3 Hours **Max. Marks : 80**

Answer any five full questions.

Q1) a) Define ordered set. Show that the set Q does not have the least upper bound property.

b) Show that every subset of a countable set A is countable.

c) State and prove Cauchy Schwarz inequality.

[6+4+6]

Q2) a) Prove that for every real $x > 0$ and every Integer $n > 0$ there is one and only one real y such that $y^n = x$.

b) Let $\{E_\alpha\}_{\alpha \in I}$ be a collection of sets E_α for each $\alpha \in I$ then prove that $\left(\bigcup_{\alpha \in I} E_\alpha\right)^c = \bigcap_{\alpha \in I}\left(E_\alpha^c\right)$

c) Let x be a metric space and $E \subset X$ then prove the following.

i) $\overline{E}$ is a closed set

ii) $E = \overline{E}$ if and only if E is closed

iii) $\overline{E} \subseteq F$ for every closed set $F \subset X$ such that $E \subset F$.

[6+4+6]

Q3) a) Show that in a metric space every k-cell is compact.

b) Let P be a non-empty perfect set in $\mathbb{R}^k$ then prove that P is uncountable.

[8+8]

Q4) a) Show that the sequence $\{x_n\}$ defined by $x_n = \left(1+\frac{1}{n}\right)^n$, where n = 1,2, ... is a bounded sequence. Further, show that x_n is a bounded monotonic sequence.

b) If $\{x_n\}$ and $\{y_n\}$ are sequences of real numbers then prove the following.

i) $\limsup_{n\to\infty}(x_n + y_n) \le \limsup_{n\to\infty}(x_n) + \limsup_{n\to\infty}(y_n)$

ii) $\liminf_{n\to\infty}(x_n + y_n) \ge \liminf_{n\to\infty}(x_n) + \liminf_{n\to\infty}(y_n)$

[8+8]

Q5) a) Suppose $a_1 \ge a_2 \ge a_3 \ge \ldots \ge a_n$ then prove that the series $\sum_{n=0}^{\infty} a_n$ converges if and only if the series $\sum_{n=0}^{\infty} 2^n a_{2^n} = a_1 + 2a_2 + 4a_4 + 8a_8 + \ldots$ converges.

b) If $\sum_{n=0}^{\infty} a_n$ converges absolutely such that $\sum_{n=0}^{\infty} a_n = \mathrm{A}$ and if $\sum_{n=0}^{\infty} b_n = \mathrm{B}$ then prove that $\sum_{n=0}^{\infty} c_n = \mathrm{AB}$, where $\sum_{k=0}^{a^n} a_k b_{n-k}$.

[8+8]

Q6) a) Suppose $\mathrm{E} \subseteq \mathrm{X}$, a metric space, p is a limit point of E, f and g are complex functions on E and $\lim_{x\to p} f(x) = \mathrm{A}, \lim_{x\to p} g(x) = \mathrm{B}.$ then prove the following.

i) $\lim_{x\to p} f(x) \pm g(x) = \mathrm{A} \pm \mathrm{B}$

ii) $\lim_{x\to p} f(x)g(x) = \mathrm{AB}$

iii) $\lim_{x\to p} \frac{f(x)}{g(x)} = \frac{\mathrm{A}}{\mathrm{B}}$

b) Define uniform continuity. Let f be a continuous mapping of a compact metric space X into a metric space Y then prove that f is uniformly continuous on X.

[8+8]

Q7) a) Show that every pathwise connected set S in $\mathbb{R}^n$ is connected.

b) If f is monotonically increasing on (a,b) then $f(c^+)$ and $f(c^-)$ both exists for each c in (a, b) and we have $\underset{a<t<c}{Sup}\, f(t) = f(c^-) \le f(c) \le f(c^+) = \underset{c<t<b}{\inf} f(t)$ further more, if $a < x < y < b$ then show that $f(c^+) \le f(y^-)$.

c) Let $f(x) = \begin{cases} x^2 \sin 1/x & if \quad x \neq 0 \\ 0 & if \quad x = 0 \end{cases}$ for $x \neq 0$ then prove that f is differentiable.

[6+6+4]

Q8) a) State and prove Cauchy's form of mean value theorem.

b) Evaluate $\lim_{x\to\infty} \frac{x^n}{e^x}$.

c) Suppose f is a continuous mapping of [a,b] into $\mathbb{R}^k$ and f is differentiable in (a,b) then prove that $|f(b) - f(a)| \le (b-a)|f'(x)|$, their exists $x \in (a,b)$.

[5+6+5]

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