MMDSC-1.2

Sl.No. : **Total No. of Pages : 3**

I Semester M.Sc. Degree Examination, June - 2025
(CBCS, SLM Scheme)
MATHEMATICS
Course - II : Real Analysis - I

Time : 3 Hours **Max. Marks : 80**

Answer any five full questions.

Q1) a) Let S is an ordered set. If S has the least upper bound property then prove that it has greatest lower bound property.

b) Show that between any two real numbers there exists a rational number, in other words Q is dense in $\mathbb{R}$.

c) Let A be a countable set, and let $A^n = \{(a_1, a_2,..., a_k,..., a_n) \mid a_k \in A$, for $1 \leq k$ be the set of all n - tuples for any $n \in \mathbb{N}$. Then prove that A^n is countable.

[4+6+6]

Q2) a) If $x, y \in \mathbb{R}$, prove that $d(x,y) = \begin{cases} 1 \text{ if } x \neq y \\ 0 \text{ if } x = y \end{cases}$ is a metric on $\mathbb{R}$.

b) If $\{G_k\}_{k=1}^{n}$ is any finite collection of open sets in a metric space, then prove that $\bigcap_{i=1}^{n} G_i$ is open.

c) If $\{K_\alpha\}_{\alpha \in I}$ is a collection of compact sets in a metric space that satisfies finite intersection condition then prove that $\bigcap_{\alpha \in I} K_\alpha$ is non empty.

[4+6+6]

Q3) a) State and prove Bolzano - Weierstrass theorem.

b) Define Cauchy sequence show that a Cauchy sequence of real numbers is conurgent.

[8+8]

Q4) a) If $\sum_{n=0}^{\infty} a_n$ is a convergent series of non - negative terms with $\{a_n\}$ decreasing then P.T $na_n \to 0$.

b) Define rearrangements. Let $\sum a_n$ be a series of real numbers which converges, but not absolutely. Suppose $-\infty \le \alpha \le \beta \le \infty$. then show that the exists a rearrangement $\sum a_n^1$ with partial sums S_n^1 such that $\lim_{n\to\infty}$ in f $S_n^1 = \alpha$ and $\lim_{n\to\infty} \sup S_n^1 = \beta$.

[4+12]

Q5) a) Let (X, d_x), (Y, d_y) be metric space $E \subseteq X$, let p be *a* limit point of E and f be a function from E to y then prove that $\lim_{x\to p} f(x) = b$ if and only if $\lim_{n\to\infty} f(x_n) = b$ for every sequence $\{x_n\}$ of points in E – $\{p\}$ which converges to p.

b) Define uniform continuity. Let f be a continuous mapping of a compact metric space X into a metric space Y then prove that f is uniformly continuous on X.

[8+8]

Q6) a) Define connectedness. Let $f : X \to Y$ be a function from a metric space X to another metric space 'Y'. Let X be a connected set. If f is continuous on X then prove that f (X) is a connected subset of Y.

b) If f is monotonically increasing on (a, b) them $f(c^+)$ and $f(c^-)$ both exists for each c in (a, b) and we have

$$\sup_{a<t<c} f(t) = f(\overline{c}) \le f(c) \le f(c^+) = \inf_{c<t<b} f(t)$$

Further more , if $a < x < y < b$, then

Prove that $f(c^+) \le f(y^-)$.

c) Define $f:\mathbb{R}\to\mathbb{R}$ by $f(x)=\begin{cases} x\sin 1/x, & \text{if } x\neq 0 \\ 1 & \text{if } x=0 \end{cases}$ them P.T f has a discontinuity at $x = 0$.

[4+8+4]

Q7) a) Let $f.g : J\to\mathbb{R}$ be differentiable at c∈J. Then prove the following holds :

i) fg is differentiable at c with $(fg)'(c)=f(c)g'(c)+g(c)f'(c)$

ii) If f is differentiable at c with $f(c)\neq 0$ then $\phi:=\frac{1}{f}$ is differentiable at c with $\phi'(c)=\frac{-f'(c)}{(f(c))^2}$.

b) State and prove inverse function theorem.

[8+8]

Q8) a) State and prove Darboux therem.

b) If $h = fg$ is a product of two functions with derivative up to order n, than prove that $h^{(n)}(x)=\sum_{k=0}^{n}\binom{n}{k}f^{(k)}(x)\,g^{(n-k)}(x)$.

[6+10]

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