MMDSC-1.2

Sl. No.

I Semester M.Sc. Degree Examination, October/November - 2025
MATHEMATICS
Real Analysis - I
(CBCS SLM Scheme)

Time : 3 Hours **Maximum Marks : 80**

Instructions to candidates :

1. ***Answer any Five full questions.***
2. ***All questions carry equal marks.***

1. a) State and prove Archimedean property. **(3+6+7)**

b) Show that between any two real numbers there exist a rational number, in other words Q is dense in $\mathbb{R}$

c) Define a countable set. Let $\{A_n\}_{n=1}^{\infty}$ be a countable collection of countable sets then prove that $\bigcup_{n=1}^{\infty} A_n$ is countable.

2. a) Let X be a metric space and $E \subset X$. If P is a limit point of E then prove that every neighbourhood of P contains infinitely many points of E. **(5+5+6)**

b) Let X be a metric space. Suppose $k \subseteq y \subseteq X$ then K is compact relative to X if and only if K is compact relative to Y.

c) If $\{G_k\}_{k=1}^{n}$ is any finite collection of open sets in a metric space, then prove that $\bigcap_{i=1}^{n} G_i$ is open.

3. a) Show that with the example a bounded monotonic Sequence is convergent. **(8+4+4)**

b) Prove that the sequence $\{x_n\}$ converges to X. If and only if every subsequence of $\{x_n\}$ converges to X.

c) Prove that every convergent sequence of real numbers is bounded.

4. a) State and prove Cauchy's first limit theorem. **(8+8)**

b) Prove that the series $\sum_{n=0}^{\infty} \frac{1}{n(\log n)^p}$ converges if p > 1 and diverges if $p \leq 1$.

[P.T.O.

5. a) Suppose $E \subseteq X$, a metric space P is a limit point of E, If f and g are complex functions on E and $\lim_{x\to p} f(x) = A$, $\lim_{x\to p} g(x) = B$. Then prove the following.

i) $\lim_{x\to p} f(x) \pm g(x) = A \pm B$

ii) $\lim_{x\to p} f(x)\, g(x) = AB$

iii) $\lim_{x\to p} \frac{f(x)}{g(x)} = \frac{A}{B}$ **(8+8)**

b) Let $A \subset \mathbb{R}$ and $f : A \to \mathbb{R}$. Show that f is continuous at the point $x_0 \in A$ if and only if for every sequence $\{x_n\}$ in A with $x_n \neq x_0 \forall n, \lim_{n\to\infty} x_n = x_0 \Rightarrow \lim_{n\to\infty} f(x_n) = f(x)$.

6. a) Define $f : \mathbb{R} \to \mathbb{R}$ *by* $f(x) = \begin{cases} x \sin 1/x, & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$ then prove that f has a discontinuity at $x = 0$. **(5+6+5)**

b) If f is a continuous mapping of a metric space X into a metric space X and if E is a connected subset of X then prove that f(E) is connected.

c) State and prove Intermediate value theorem.

7. a) State and prove inverse function theorem. **(8+8)**

b) Let f.g : $J \to \mathbb{R}$ be differentiable at $C \in J$ then prove the following.

i) f + g is differentiable at C with $(f+g)'C = f'(C) + g'(C)$

ii) αf is differentiable at C with $(\alpha f)'C = \alpha f'(C)$

iii) fg is differentiable at C with $(fg)'C = f(C)g'(C) + g(C)f'(C)$

8. a) State and prove Darboux's theorem. **(4+9+3)**

b) If h = fg is a product of two functions with derivative upto order n, then prove that

$$h^{(n)}(x) = \sum_{k=0}^{n} \begin{bmatrix} n \\ k \end{bmatrix} f^{(k)}(x)\, g^{(n-k)}(x)$$

c) Find $\lim_{x\to 0} \frac{\tan x - x}{x - \sin x}$.
