MMDSC-1.3

Sl. No.

I Semester M.Sc. Degree Examination, December - 2023

MATHEMATICS

Complex Analysis - I

(CBCS Scheme)

Time : 3 Hours **Maximum Marks : 80**

***Instructions to Candidates*:**

1) Answer any **Five** full questions.
2) All questions carry **Equal** marks.

1. a) Prove the following elementary properties of conjugate of a complex number. **(8+8)**

i. $\overline{(\bar{z})} = z$

ii. $\overline{z_1 + z_2} = \bar{z_1} + \bar{z_2}$

iii. $\overline{(z_1 z_2)} = \bar{z_1}.\bar{z_2}$

iv. $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z_1}}{\bar{z_2}}$ $(where,\ z_2 \neq 0)$

b) For $z \neq 0$ and $w \neq 0$, demonstrate that $|z+w| = |z|+|w|$ is true if and only if $w=tz$ for some $t>0$.

2. a) Let z_1 and z_2 be two complex numbers, then prove, **(8+4+4)**

i. $\arg(z_1 z_2) = \arg z_1 + \arg z_2$

ii. $\arg\left(\frac{z_1}{z_2}\right) = \arg z_1 - \arg z_2 \quad , z_2 \neq 0$

b) State and prove triangular inequality.

c) Show that

i. $\big|\,|z_1| - |z_2|\,\big| \leq |z_1 + z_2|$

ii. $\left|\frac{z_1 - z_2}{1 - \bar{z_1} z_2}\right| < 1$ if $|z_1|<1$ and $|z_2|<1$

[P.T.O.

3. a) If a sequence $\{a_n\}_{n=1}^{\infty}$ of sequence of complex numbers converges to L. Prove that the arithmetic sum sequence $\left\{\frac{a_1 + ... + a_n}{n}\right\}_{n=1}^{\infty}$ also converges to L. **(8+4+4)**

b) Check whether the limit, $\lim_{z\to 0}\frac{\overline{z}}{z}$ exists or not.

c) Evaluate the limit of $f(z)$ at z=0 if it exists, where $f(z)=\frac{\mathrm{Re}\, z}{|z|}$.

4. a) Find the derivative of $w = f(z) = 3z^2 + 4iz - 5 + i$ at $z = z_0$. **(6+10)**

b) State and prove sufficient conditions for $f(z)$ to be analytic.

5. a) Derive polar form of Cauchy-Riemann equations. **(10+6)**

b) Prove that $f(z)=\sin x.\cos hy + i.\cos x.\sin hy$ is differentiable at every point.

6. a) State and prove Ratio Test. **(6+10)**

b) State and prove Cauchy-Hadamard theorem.

7. a) State and prove Abel's limit theorem. **(8+8)**

b) State and prove Cauchy's Integral formula.

8. a) If $f(z)$ is analytic in a region bounded by two simple closed curves $\mathbb{C}$ and $\mathbb{C}_1$ (where C_1 lies inside C) and on these curves. **(8+8)**

Prove that , $\int_C f(z)dz = \int_{C_1} f(z)dz$

Where C and C_1 are both traversed in the positive sense related to their interiors.

b) Develop the functions $f(z) = z^{-1}$ and $g(z) = z^{-2}$ in Laurent series in the annulus, $D = \{z : 1 < |z - i| < \infty\}$.
