MMDSC-1.3

Sl.No. :

Total No. of Pages : 3

I Semester M.Sc. Degree Examination, June/July - 2024

(CBCS, SLM Scheme)

MATHEMATICS

Complex Analysis - I

Time : 3 Hours **Max. Marks : 80**

Instruction: ***1)*** **Answer any five questions.**

2) All questions carry equal marks.

Q1) a) For a given z = x + iy, prove that $\sqrt{z} = \pm\left[\sqrt{\frac{|z|+x}{2}} + is\ gn(y)\sqrt{\frac{|z|-x}{2}}\right]$,

where s gn (y) = $\begin{cases} 1, if\ \ y \geq 0 \\ -1, if\ \ y < 0. \end{cases}$

b) State and prove Lagrange's identity.

c) Prove the centroid of the triangle whose vertices are z_1, z_2, z_3 is $\frac{z_1+z_2+z_3}{3}$.

[5+8+3]

Q2) a) Establish a one-one correspondence between the set of all points on the complex plane and the set of all points on the Riemann sphere.

b) Show that all circles that pass through a and $\frac{1}{\overline{a}}$ intersect the circle $|z| = 1$ at right angles. **[8+8]**

Q3) a) Does there exist an analytic function whose real part is $u = xy^2$? If so, find the analytic function $f = u + iv$. If not, explain the reason why it is not.

b) State and prove Weierstrass M-test.

[8+8]

Q4) a) Prove that every power series represents an analytic function inside its circle of convergence.

b) Let R_1 and R_2 be radii of convergence of the power series $f(z) = \sum_{n=0}^{\infty} a_n z^n$ and $g(z) = \sum_{n=0}^{\infty} b_n z^n$

respectively. Find the radii of convergence of $f(z) + g(z)$ and $f(z).g(z)$.

[8+8]

Q5) a) Show that $w = \dfrac{z-1}{z+1}$ maps the imaginary axis in the Z-plane onto the circle $|w|=1$ in the W-plane.

b) Prove that any bilinear transformation preserves cross ratio.

c) Find the bilinear transformation that maps $-1, 1, \infty$ to the points -i, -1, i. **[6+6+4]**

Q6) a) Prove that the line integral $\int_y Pdx + Qdy$, defined in Ω depends only on the end points of γ if and only if there exists a function $U(x, y)$ in Ω with the partial derivatives $\dfrac{\partial U}{\partial x} = P, \dfrac{\partial U}{\partial y} = Q$.

b) If C is a curve given by $y = x^3 - 3x^2 + 4x - 1$ joining the points (1,1) and (2,3), find the value of $\int_C (12z^2 - 4iz)dz$.

[8+8]

Q7) a) State and prove Cauchy's theorem for rectangle.

b) Evaluate i) $\int_{|z|=2} \frac{dz}{z^2+1}$

ii) $\int_{|z|=1} \frac{e^z}{z} dz$

[10+6]

Q8) a) State and prove Morera's therorem.

b) Expand $f(z) = \frac{z-1}{z+1}$ as Taylor's series about

i) the point z = 0.

ii) the point z=1.

[10+6]