MMDSC-1.3

Sl.No. :

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I Semester M.Sc. Degree Examination, May/June - 2026
(SLM Scheme, CBCS)
MATHEMATICS
Course - III : Complex Analysis - I

Time : 3 Hours **Max. Marks : 80**

Instruction : ***Answer any five full questions.***

Q1) a) Let z_1 and z_2 are two complex numbers, then prove that

i) $\text{org}(z_1 z_2) = \text{org } z_1 + \text{org } z_2$

ii) $\text{org}\left(\frac{z_1}{z_2}\right) = \text{org } z_1 - \text{org } z_2, z_2 \neq 0.$

b) Prove that $\left|\frac{z_1 - z_2}{1 - \bar{z}_1 z_2}\right| = 1$ if either $|z_1| = 1$ or $|z_2| = 1$. What exception must be made if $|z_1| = |z_2| = 1$?

c) Evaluate $\lim_{n\to\infty}\left\{\frac{3n + 7ni}{2n + 5i}\right\}$.

[6+5+5]

Q2) a) Prove that the sequence $\{a_n\}_{n=1}^{\infty}$ of sequence of complex numbers converges to L, then the arithmetic sum sequence $\left\{\frac{a_1 + \ldots + a_n}{n}\right\}_{n=1}^{\infty}$ also converges to L.

b) Define

i) Open disk

ii) Closed disk

iii) Punctured disk

c) Let $f : A \to \mathbb{C}$ and $g : A \to \mathbb{C}$ be two functions. Suppose $\lim_{z \to z_0} f(z) = w_0$ and $\lim_{z \to z_0} g(z) = w^1$, then prove that

i) $c \lim_{z \to z_0} f(z) = \lim_{z \to z_0} c\, f(z)$

ii) $\mathrm{Re}(z) \to \mathrm{Re}(w_0)$ and $\mathrm{Im}(z) \to \mathrm{Im}(z_0)$ as $z \to z_0$.

[6+6+4]

Q3) a) State and prove cauchy - Reimann equation in polar form.

b) If $f(z)$ and $\overline{f(z)}$ are both analytic in a region D, then prove that *f*(z) is a constant in that region.

c) Prove that $u = 2x - x^3 + 3xy^2$ is harmonic and find its harmonic conjugate, also find the corresponding analytic function.

[6+4+6]

Q4) a) State and prove Cauchy - Hadamard theorem.

b) State and prove Cauchy's Integral formula.

[8+8]

Q5) a) If C is the curve $y = x^3 - 3x^2 + 4x - 1$ joining points (1, 1) and (2, 3), find the value of $\int_C (12z^2 - 4iz)\, dz$.

b) If z_1 and z_2 are any two points in R (a simply connected region) then $\int_{z_1}^{z_2} f(z)\, dz$ is independent of the path in R joining z_1 and z_2.

c) Evaluate $\int_C \frac{z^3 - 2z + 1}{(z - i)^2}\, dz$, where C is the circle $|z| = 2$.

[6+5+5]

Q6) a) State and prove cauchy's theorem for disk.

b) Find and classify the isolated singularities of $f(z)=\frac{\sin z}{z}$.

[10+6]

Q7) a) State and prove weierstrass theorem.

b) Discuss the convergence of $\sum_{n=0}^{\infty} z^n$.

[12+4]

Q8) a) Develop the functions $f(z)=z^{-1}$ and $g(z)=z^{-2}$ in Laurent series in the annulus, $\mathrm{D}=\{z:1<|z-i|<\infty\}$.

b) Determine the Taylor's series expansion of $f(z)=\cos^2 z$ about the origin.

[10+6]