Sl.No. :

Total No. of Pages : 2

I Semester M.Sc. Degree Examination, June-2025
(CBCS, SLM Scheme)
MATHEMATICS
Course - III: Complex Analysis - I

Time : 3 Hours **Max. Marks : 80**

Instruction : Answer any five full questions.

Q1) a) Show that $|z_1| - |z_2| \leq |z_1 + z_2|$.

b) Prove that $\left|\frac{z_1 - z_2}{1 - z_1 z_2}\right| = 1$ if either $|z_1| = 1$ or $|z_2| = 1$. What exception must be made if $|z_1| = |z_2| = 1$?

c) Evaluate $\lim_{z \to e^{i\frac{\pi}{3}}} \left(z - e^{i\frac{\pi}{3}}\right) \frac{z}{(z^3 + 1)}$.

[5+5+6]

Q2) a) State and prove Cauchy-Hadamard theorem.

b) Show that the principal values of z^i remain bounded for all values of z.

c) Prove that Every compact subset of C is closed.

[6+5+5]

Q3) a) State and prove Cauchy-Riemann equation polar form.

b) Find the derivative of $w = f(z) = 3z^2 + 4iz - 5 + i$ at $z = z_0$.

c) Find the analytic function $f(z) = u + iv$ given that $u - v = e^x (\cos y - \sin y)$.

[6+5+5]

Q4) a) Prove that $f(z) = \sin x \cosh y + i \cos x \sinh y$ is differentiable at every point.

b) Show that an analytic function with constant modulus is constant.

c) State and prove Ratio test.

[6+5+5]

Q5) a) Evaluate $\int_c |z|\, z dz$, where c is the closed curve consisting of the upper semi circle $|z| = 1$ and the segment $-1 \le x \le 1$.

b) Find the singularities of $\frac{z}{e^{\frac{1}{z}} - 1}$.

c) Find and classify the isolated singularities $\frac{\cos z}{z^2 + 1} + 4z$.

[6+5+5]

Q6) a) State and prove Cauchy's theorem for rectangles.

b) Evaluate $\int_c \frac{z^3 - 2z + 1}{(z - i)^2} dz$, where c is the circle $|z| = 2$.

[8+8]

Q7) a) State and prove weierstrass theorem.

b) Discuss the convergence of the geometric series $\sum_{n=0}^{\infty} z^n$.

c) Discuss the Taylor series expansion of the function $f(z) = \log z$ about the point $z_0 = -1 + i$.

[6+5+5]

Q8) a) Expand the function $f(z) = e^{\frac{1}{z}}$ in a laurent series in the ring $D = \{z : 0 < |z| < \infty\}$.

b) Expand $f(z) = \frac{2z}{z^2 - 1}$ in power series centered around $z_0 = i$.

c) Evaluate $\int_0^{2\pi} e^{e^{i\theta}} d\theta$.

[5+6+5]

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