MMDSC-1.3

Sl. No.

I Semester M.Sc. Degree Examination, October/November - 2025
MATHEMATICS
Complex Analysis - I
(CBCS SLM Scheme)

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

Answer any Five full questions.

1. a) Prove the following : **(6+5+5)**

i) $\big||z_1|-|z_2|\big| \le |z_1+z_2|$

ii) $\left|\dfrac{z_1-z_2}{1-\vec{z}_1 z_2}\right| < 1$ if $|z_1| < 1$ and $|z_2| < 1$.

b) For any two complex numbers z_1 and z_2, prove that $|z_1+z_2| \le |z_1|+|z_2|$.

c) Prove that every convergent sequence is bounded.

2. a) Prove that a set $A \subset \mathbb{C}$ is closed if and only if $A = \overline{A}$. That is, if and only if A contains all its boundary points. **(6+5+5)**

b) Show that intersection of arbitrary collection of open sets need not be open.

c) Show that $\lim_{Z \to 1-i} \overline{Z}^2 = (1+i)^2$.

3. a) State and prove Cauchy-Riemann equations in Cartesian form. **(6+5+5)**

b) Prove that *f (z) = sin x cos hy + i cos x sinhy* is differentiable at every point.

c) Show that the principal values of z^i remain bounded for all values of z.

4. a) State and prove Abel's limit theorem. **(8+8)**

b) State and prove Ratio Test.

5. a) Evaluate $\int_{(0,3)}^{(2,4)} (2y+x^2)\,dx + (3x-y)\,dy$ along the parabola $x = 2t, y = t^2+3$. **(6+5+5)**

b) Find the singularities of $\sin\left(\dfrac{1}{1-z}\right)$

c) If f(z) be analytic in a region bounded by two simple closed curves C and C_1 and on these curves. Then prove that $\int_C f(z)\,dz = \int_{C_1} f(z)\,dz$ Where C and C_1 are both transversed in the positive sense related to their interiors.

[P.T.O.

6. a) State and prove Liouvilles Theorem. **(8+8)**

b) Evaluate $\int_C \frac{dz}{(z-i)(z+i)}$ where C is the circle $|z-i|=1|$

7. a) Expand f(z) = logz in a Taylor series about the point z_0=1. **(6+6+4)**

b) Determine the Laurent series representation of $f(z) = (z - 1)^{-3}\ SinZ$ in the ring $D = \{z : 0 < |z-1| < \infty\}$

c) Discuss the Convergence of $\sum_{n=1}^{\infty} \frac{z^n}{2^n}$

8. a) State and prove Laurent's Theorem. **(10+6)**

b) Show that $\sum_{n=0}^{\infty} \frac{z(z+1)...(z+n-1)}{n^n}$ defines an entire function.
