MMDSC-1.4

Sl. No.

I Semester M.Sc. Degree Examination, December - 2023
MATHEMATICS
Ordinary Differential Equations
(CBCS Scheme)

Time : 3 Hours **Maximum Marks : 80**

***Instructions to candidates*:**

1) ***Answer any Five full questions.***
2) ***All questions carry equal marks.***

1. a) Define ordinary differential equation and give two examples.

b) Find particular solution of $y'' - y = 0$, given condition y(0)=1 : $y'(0)=1$.

c) If $y_1(x)$ and $y_2(x)$ are any two solution of $y'' + P(x)y' + Q(x)y = 0$ on [a,b] then prove that, their wronskian $W(x) = W(y_1, y_2)$ is either identically zero or never zero. **(3+5+8)**

2. a) Define wronskian. Find the wronskian of the independent solutions $\phi_1(x)$ *and* $\phi_2(x)$ of $y'' + k^2 y = 0$; where, K is a constant satisfying the initial conditions $\phi_1(0) = \sin\alpha, \phi_1^1(0) = -\cos\alpha, \phi_2(0) = \cos\alpha, \phi_2^1(0) = \sin\alpha, (\alpha = \text{constant})$.

b) Solve by the method of variation of parameter: $x^2y'' - xy' - 3y = x^3$. **(8+8)**

3. a) Obtain Legendre's polynomial $P_n(x)$ using series solution of Legendre's differential equation.

b) Prove that,

i) $J_{-n}(x) = (-1)^n . j_n(x)$

ii) $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\, cosx$ **(10+6)**

[P.T.O.

4. a) Express $x^3 + 2x^2 - 4x + 5$ in terms of Legendre's Polynomial.

b) Find the general solution for the following homogeneous linear system of differential equations.

$$\frac{dx_1}{dt} = x_1 + x_2 - x_3$$
$$\frac{dx_2}{dt} = 2x_1 + 3x_2 - 4x_3.$$
$$\frac{dx_3}{dt} = 4x_1 + x_2 - 4x_3$$

(6+10)

5. a) Define Critical point, trajectories, saddle point, stable and unstable critical points of a system of differential equations.

b) Find all critical points and their nature and stability of the non-linear system $\frac{dx}{dt} = 1 - xy$; $\frac{dy}{dt} = x - y^3$. **(6+10)**

6. a) If $q(x) < 0$ and $v(x)$ is a non-trivial solution of $u'' + q_r(x)u = 0$, then prove that $v(x)$ has atmost one zero.

b) State and prove sturm's comparison theorem. **(8+8)**

7. a) Find the eigen value and eigen functions of $y'' + \lambda y = 0$; $y(0) = y(\pi) = 0$.

b) Solve the given differential equation, $(1 - x^2)y'' - xy' + n^2 y = 0$ in normal form **(8+8)**

8. Obtain eigen value and eigen functions for Chebyshev equation and Lagguerre equation.

(16)
