MMDSC-1.4

Sl.No. : **Total No. of Pages : 3**

I Semester M.Sc. Degree Examination, June/July - 2024

(CBCS , SLM Scheme)

MATHEMATICS

Ordinary Differential Equations

Time : 3 Hours **Max. Marks : 80**

Instructions: ***1) Answer any five full questions.***
2) All questions carry equal marks.

Q1) a) Explain the method of variation of parameters for solving $\frac{dy}{dx} + p(x)y = Q(x)$.

b) If $y_1(x)$ and $y_2(x)$ are any two solution of $Y''+p(x)y'+Q(x)y=0$ on $[a,b]$ then their Wronskian $w(x) = w(y_1, y_2)$ is either identically zero or never zero.

[6+10]

Q2) a) Find the general solution and particular solution for $y_1=e^x$, $y_2=e^{-2x}$, $y''+y'-2y = 0$ with $y(0) = 8$, $y'(0) = 2$.

b) Verify Liouville's formula for $x^3y''' - 3x^2y''+6xy' - 6y = 0$

[8+8]

Q3) a) If x is one solution of $x^2y''-xy'+y = 0$, $x \neq 0$. Find the other solution.

b) Find the adjoint differntial equation to $x^2y''+7xy'+8y = 0$.

c) Verify Legrange's identity for the equation $y''+y = 0$

[6+4+6]

Q4) a) State and prove Sturm's comparison theorem.

b) Prove that $P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n$.

[8+8]

Q5) a) Show that $J_{-n}(x) = (-1)^n J_n(x)$.

b) Express x^3+2x^2-4x+5 interms of Legendre's polynomial.

c) Prove that $L'_n(x) = L'_{n-1}(x) - L_{n-1}(x)$.

[6+4+6]

Q6) a) Find the General solution for the hamogenous Linear system of equation

$$\frac{dx}{dt} = \begin{pmatrix} 1 & 1 & -1 \\ 2 & 3 & 4 \\ 4 & 1 & -4 \end{pmatrix} x.$$

b) Find all the critical points, and their nature, stability of the non -linear system

$$\frac{dx}{dt} = 8x - y^2, \quad \frac{dy}{dt} = -6y + 6x^2.$$

[10+6]

Q7) a) State and prove picard's theorem for existence and uniqueness.

b) Find the Eigen Values and Eigen functions of $y''+\lambda y = 0$; $y(0) = 0 = y(\pi)$.

[8+8]

Q8) a) Solve by Green's method for the differential equation $\frac{d^2y}{d^2x^2}+y=1$ with boundary conditions y(0)+Y'(0) = 0 and $y\left(\frac{\pi}{2}\right)+y'\left(\frac{\pi}{2}\right)=0$.

b) Using the picard's method to obtain the fourth approximation to the solution of $\frac{dy}{dx}=x+y, y(0)=1$ and hence find y at $x = 0.1, 0.2$.

[10+6]

