MMDSC-1.4

Sl.No. : **Total No. of Pages : 2**

I Semester M.Sc. Degree Examination, May/June - 2023

(CBCS, SLM Scheme)

MATHEMATICS

Ordinary Differential Equations

Time : 3 Hours **Max. Marks : 80**

Instructions: ***1) Answer any five full questions.***
2) All questions carry equal marks.

Q1) a) Define linear ordinary differential equation and non-linear ordinary differential equation. Give examples for each.

b) Differentiate between Homogeneous and Non-homogeneous second order linear ordinary differential equations.

c) If $y_1(x)$ and $y_2(x)$ are any two solutions of $y'' + P(x)y' + Q(x)y = 0$ on $[a, b]$ then prove that, their wronskian $W(x) = W(y_1, y_2)$ is either identically zero or never zero.

[4+4+8]

Q2) a) Explain briefly on method of variation of parameters.

b) Find the particular solution for non-homogeneous equation $y'' + y = \tan x$ by the method of variation of parameters.

[10+6]

Q3) a) Obtain Legendre's polynomial $P_n(x)$ using series solution of Legendre's differential equation.

b) Prove that,

i) $J_{-n}(x) = (-1)^n J_n(x)$

ii) $J_n(-x) = (-1)^n J_n(x) = J_{-n}(x)$

[10+6]

Q4) a) Express $x^3 + 2x^2 - 4x + 5$ in terms of Legendre's polynomial.

b) If 'α' and 'β' are two distinct roots of $J_n(x) = 0$, then prove that,

$$\int_0^1 x.J_n(\alpha x)\,J_n(\beta x)dx = \left\{ \begin{array}{l} 0 \text{ if } \alpha \neq \beta \\ \frac{1}{2}\left[J_n(\alpha)\right]^2 \\ \frac{1}{2}\left[J_{n+1}(\alpha)\right]^2 \end{array} \right\} \text{if } \alpha = \beta$$

[4+12]

Q5) a) Find the general solution of following homogeneous linear equations:

$$\frac{dX}{dt} = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 3 & 1 \\ -3 & -6 & 6 \end{bmatrix} X$$

b) Define critical point trajectories, saddle point, stable and unstable critical points of a system of differential equations.

[10+6]

Q6) a) Find all critical points and their nature and stability of the non-linear system $\frac{dx}{dt} = 1 - xy, \frac{dy}{dt} = x - y^3$.

b) If $u(x)$ is a non-trivial solution of $u'' + q(x)u = 0$ and $q(x) > 0$ and $\int_1^\infty q(x)dx = \infty$ then prove that $u(x)$ has infinitely many zeroes on the positive x - axis.

[8+8]

Q7) a) State and prove sturm's comparision theorem.

b) Obtain Eigen value and Eigen function for Hermite equation.

[12+4]

Q8) a) Define Eigen function expansion and mean square error.

b) State and prove orthogonality property of Eigen functions.

[4+12]

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