MMDSC - 2.1

Sl.No. : **Total No. of Pages : 2**

II Semester M.Sc. Degree Examination, June - 2023
(CBCS, SLM Scheme)
MATHEMATICS
Algebra - II

Time : 3 Hours **Max. Marks : 80**

Instructions: ***1) Answer any five full questions.***
2) All questions carry equal marks.

Q1) a) Define local ring. If $n > 1$ is an integer, then prove that $\mathbb{Z}_n$ is a local ring if and only if n is a power of a prime.

b) Prove that $N(A) = \bigcap_{P\text{-Prime}} P$.

c) Define the Jacobson radical of a ring. Find the Jacobson radical of the ring of integers.

[6+8+2]

Q2) a) If I is an ideal of a ring A. Then prove that

i) r (I) is an ideal of A containing I.

ii) r (I) is the intersection of all prime ideals which contain I.

b) Prove that N(A) is an ideal of A and also prove that $\frac{A}{N(A)}$ has no non zero nilpotents, where N(A)$\rightarrow$ Nilradical of A.

[8+8]

Q3) a) State and prove Nakayama limma.

b) State and prove Schur's limma.

[8+8]

Q4) a) Show that M is a finitely generated A-module if and only if M is isomorphic to a quotient of a finitely generated fru A-module.

b) State an prove Hilbert basis theorem.

[8+8]

Q5) a) If N is a submodule of M. then prove that M is Noetherian if and only of N and M/N are Noetherian.

b) Show that a module is of finitel length if and only if it is both Artinian and Noetherian.

[8+8]

Q6) a) Let K be an finite extersion of field F and L be a finite extersion of K. Then Prove that L is a finite extersion of F and [L:F] = [L:K] [K:F].

b) Find the degree of extension of $Q\left[\sqrt{2},\sqrt{3}\right]$ over Q.

c) Let K,L be field extensions of F & $\alpha \in K$, $\beta \in L$ be algebraic over F. Then prove that there exist an F isomorphism $\sigma : F(\alpha) \rightarrow F(\beta)$ satisfying $\sigma(\alpha) = \beta$ if and only if the minimal polynomial of α & β over F are same.

[5+4+7]

Q7) a) If a real number α is constructible, then prove that there exists a finite sequence of fields $Q = F_1 \subseteq F_2 \subseteq ....\subseteq F_n \subseteq F_{n+1} = K$ satisfying

i) K is a subfield of $\mathbb{R}$

ii) $\alpha \in K$

iii) $[F_{i+1} : F_i] = 2$ for $1 \leq i \leq n$.

Also prove that [K : Q] is power of 2.

b) If K is any field then prove that characteristic of K is prime.

c) Prove that an element α which is any extension of F, is a multiple. root of $f(x)$ if and only if α is a root of both $f(x)$ and $f^1(x)$.

[7+3+6]

Q8) a) Let ch F = P > 0. Then prove that F is perfect if and only if every element of F has a pth root in F.

b) Let K/F be a finite separable extension... Then prove that $K=F(\alpha)$ for some $\alpha \in K$

c) Prove that K is a normal extension of F is and only if K is the splitting field of some polynomial over F.

[4+6+6]

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